Name Mrs. R date 8/9/13

Unit 1 Notes

1.1 Properties of Numbers

Real Numbers: Numbers that are used in the real world and can be represented by a point on the number line.

Rational Numbers: A number that can be expressed as a fraction, or is a repeating or terminating decimal. This includes natural numbers, whole numbers, and integers.

Irrational Numbers: A number that is a decimal that does not repeat, and does not end. (cannot be written as a fraction)

Real

$\frac{1}{2}$, .3, $\sqrt{36}$

Rational

Irrational + π, $\sqrt{5}$, $\sqrt{2}$

Integers + …-2, -1, 0, 1, 2…

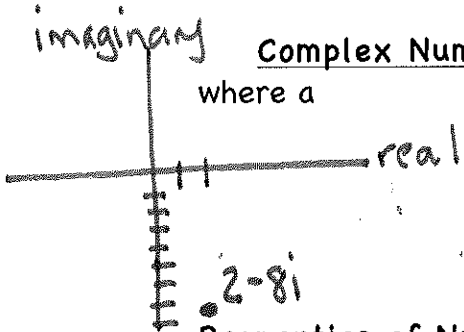
Whole + 0, 1, 2, 3…

Natural + 1, 2, 3…

$\sqrt{-4} = \sqrt{4 \cdot -1} = \sqrt{4} \cdot \sqrt{-1} = 2i$

Pure Imaginary Numbers- non- real numbers that come from square roots of negative real numbers (3i, -5i, i$\sqrt{2}$)

The letter I - the imaginary unit ($i^2 = -1$) $\sqrt{-1} = i$



Complex Number- any number that can be written in the form **a + bi,** where a and b are real numbers, and i is the imaginary unit. a is called the real part, b is called the imaginary part.

$\underset{\text{real}}{2} - \underset{\text{imaginary}}{8i}$

Properties of Numbers

Switch order

parenthesis

PROPERTY	ADDITION	MULTIPLICATION
COMMUTATIVE	A + B = B + A	A • B = B • A
ASSOCIATIVE	A + (B + C) = (A+B) + C	A • (B • C) = (A• B) • C
IDENTITY	A + 0 = A	A • 1 = A
INVERSE	A + (-A) = 0	A • 1/A = 1 (A ≠ 0)
DISTRIBUTIVE	A (B + C) = AB + AC	A(B – C) = AB – AC

Ex. 1 Classify the sets of numbers.

a. $\sqrt{144} = 12$

Real
rational
integer
whole
natural

b. $2\sqrt{5}$

Real
Irrational

c. 2π

Real
irrational

d. $-3i$

Pure imaginary

e. 0

Real
Rational
Integer
whole

f. -18

Real
Rational
Integer

g. $2 + 4i$

Complex number

Ex. 2 Identify the properties

a. $(5+7)+8 = 8+(5+7)$

Commutative prop.

b. $3(4x) = (3 \cdot 4)x$

associative prop.

Ex. 3 Identify the Additive and Multiplicative Inverses

	A	M
a. 1.25	-1.25	$\frac{4}{5}$
b. $-1\frac{3}{4}$	$1\frac{3}{4}$	$-\frac{4}{7}$

$1\frac{1}{4}$

$1+\frac{1}{4}$

$\frac{4}{4}+\frac{1}{4}$

$=\frac{5}{4}$

$-\left(1+\frac{3}{4}\right)$

$-\left(\frac{4}{4}+\frac{3}{4}\right)$

$-\frac{7}{4}$

C.W. name set(s) numbers belong to:

p15

21. $\sqrt{121} = 11$ Real, rational, integer, whole natural

23. $\sqrt{10}$ Real, irrational

26. $\frac{3\pi}{2}$ Real, irrational

PEMDAS

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1.2Expressions and Formulas

Variables: are symbols, usually letters, used to represent unknown quantities

Algebraic Expressions: A mathematical phrase that contains numbers and a variable (does not have an equal sign)

Algebraic Equation: A mathematical sentence that contains numbers and a variable (has an equal sign)

Ex. 1 $[2(10-4)^2+3] \div 5$

$[2(6)^2+3] \div 5$

$[2(36)+3] \div 5$

$[72+3] \div 5$

$75 \div 5$

$\boxed{15}$

Ex. 2 $x^2 - y(x+y)$ if $x=3$, $y=15$

$3^2 - 15(3+15)$

$3^2 - 15(18)$

$9 - 15(18)$

$9 - 270$

$\boxed{-261}$

$$\begin{array}{r} 15 \\ \times\ 18 \\ \hline 120 \\ 15\ \ \\ \hline 270 \end{array}$$

Ex. 3 if $a=2$, $b=-4$, $c=-3$

$$\frac{a^3+2bc}{c^2-5}$$

$$\frac{2^3+2(-4)(-3)}{(-3)^2-5}$$

$$\frac{8+2(-4)(-3)}{9-5}$$

$$\frac{8+24}{9-5}$$

$$\frac{32}{4} = \boxed{8}$$

$(-3)(-3)$

Ex. 4 Area of a trapezoid is 340 in² The height of the trapezoid is 10 in. and one of the bases is 16 in. Find the measure of the other base.

$A = \frac{1}{2}h(b_1+b_2)$

$340 = \frac{1}{2} \cdot 10(16+b_2)$

$\frac{340}{5} = \frac{5(16+b_2)}{5}$

$68 = 16 + b_2$

$-16 \quad -16$

$52 = b_2$

→

$$340 = 5(16 + b_2)$$

$$340 = 80 + 5b_2$$

$$-80 \quad -80$$

$$\frac{260}{5} = \frac{5}{5}b_2$$

$$52 = b_2$$

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1.3Monomials

Monomial- one termed polynomial with no variable in the denominator ex. 1, -2x, 4xy

Constants- monomials that have no variable (just a number)

Coefficient- The number in front of a variable (3x - coefficient would be 3)

Degree- in a monomial it's the sum of the exponents of its variables. In a polynomial it is the largest exponent of the monomials

Power- the exponent

****Negative Exponents***** $x^{-a} = \frac{1}{x^a}$, $\frac{1}{x^{-b}} = x^b$

Important Imaginary Number Exponential Equivalency Chart

Imaginary Number	Equivalency
i	i
i^2	-1
i^3	$-i$
i^4	1

Ex. 1 Simplify Expressions with Multiplication

a. $(3x^3y^2)(-4x^2y^4)$

$(3)(-4)(x^3)(x^2)(y^2)(y^4)$

xxx xx yy yyyy

$-12x^5y^6$

b. $-2i \cdot 7i$

$(-2)(7)\, i\, i$

$-14i^2 = (-14)(-1)$

$\boxed{14}$

Ex. 2 Simplify Expressions with Division

FFOO
fancy form of one

a. $\frac{x^5}{x^9} = \frac{\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}\cancel{x}\cancel{x}\cancel{x}xxxx}$

$= \frac{1}{x^4}$

b. $\frac{34x^5y^2}{26x^3y^9}$

$\frac{\cancel{2}\cdot 17\,\cancel{x}\cancel{x}\cancel{x}xx\,\cancel{y}\cancel{y}}{\cancel{2}\cdot 13\,\cancel{x}\cancel{x}\cancel{x}\,yyyyyyyyy}$

$= \frac{17x^2}{13y^7}$

Ex. 3 Simplify Expressions with Powers

a. $(a^3)^6 = a^3 a^3 a^3 a^3 a^3 a^3$

$\boxed{a^{18}}$

b. $(-2p^3r^2)^5$

$(-2)^5(p^3)^5(r^2)^5$

$\boxed{-32p^{15}r^{10}}$

c. $\left(\frac{-3x}{y}\right)^4 = \frac{(-3)^4(x^4)}{y^4}$

$\boxed{\frac{+81x^4}{y^4}}$

d. $\left(\frac{a^4}{4a^2b}\right)^{-3} = \left(\frac{4a^2b}{a^4}\right)^3$

$\left(\frac{4b}{a^2}\right)^3 = \frac{4^3b^3}{(a^2)^3} = \boxed{\frac{64b^3}{a^6}}$

e. $i^{45} = i^{44} \cdot i^1$

$(i^4)^{11} \cdot i = (1)^{11} \cdot i = 1 \cdot i = \boxed{i}$

f. $i^{24} = (i^4)^6 = (1)^6 = \boxed{1}$

Ex. 4 Simplify Expressions Using Several Properties

$\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4 = \left(\frac{-2\cancel{x^n}\cancel{x^n}x^n}{\cancel{x^n}\cancel{x^n}yyy}\right)^4 = \left(\frac{-2x^n}{y^3}\right)^4$

$\frac{(-2)^4(x^n)^4}{(y^3)^4} = \boxed{\frac{+16x^{4n}}{y^{12}}}$

$a.bc \times 10^n$

Scientific Notation- a x 10^n, where a has to be between $1 \leq a < 10$
Standard Notation- when the number is written out in expanded form

Ex. 5 Multiply Numbers in Scientific Notation

a. $(4 \times 10^3)(6 \times 10^9) = (4)(6) \times (10^3)(10^9)$
$= 24 \times 10^{12} = \underbrace{(2.4 \times 10^1) \times 10^{12}} = \boxed{2.4 \times 10^{13}}$

b. $(2.7 \times 10^{-2})(3 \times 10^6) = (2.7)(3) \times (10^{-2})(10^6)$

$\boxed{8.1 \times 10^4}$

Ex. 6 Divide Numbers in Scientific Notation

a. $$\frac{4 \times 10^6}{2 \times 10^3}$$

$$\left(\frac{4}{2}\right) \times \left(\frac{10^6}{10^3}\right)$$

$$\boxed{2 \times 10^3}$$

$$\boxed{2.00 \times 10^3}$$

b. $$\frac{3 \times 10^4}{5 \times 10^2}$$

$$\left(\frac{3}{5}\right) \times \left(\frac{10^4}{10^2}\right)$$

$$.6 \times 10^2$$

$$\underbrace{(6.0 \times 10^{-1}) \times 10^2}$$

$$\boxed{6.0 \times 10^1}$$

$\frac{3}{5}$: $5\overline{)3.00}$ = $.6$

$\underline{30}$

$$\boxed{i = \sqrt{-1}}$$

$$i^2 = \left(\sqrt{-1}\right)^2 = -1$$

$$i^3 = i^2 \cdot i = -1 \cdot i = -i$$

$$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$$

$$= \left(i^2\right)^2 = (-1)^2 = 1$$

$$i^{25} = i^{24} \cdot i^1$$

$$= \left(i^4\right)^6 \cdot i$$

$$= (1)^6 \cdot i$$

$$= 1 \cdot i = i$$

$$i = \sqrt{-1} = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$i^5 = i^4 \cdot i = 1(i) = i$$

$$i^6 = i^4 \cdot i^2 = 1(-1) = -1$$

$$i^7 = i^4 \cdot i^3 = 1(-i) = -i$$

$$i^8 = i^4 \cdot i^4 = (1)(1) = 1$$

$$i^9 = i^4 \cdot i^4 \cdot i = (1)(1) \cdot i = i$$

$$i^{10} = i^4 \cdot i^4 \cdot i^2 = (1)(1)(-1) = -1$$

$$i^{11} = i^4 \cdot i^4 \cdot i^3 = (1)(1)(-i) = -i$$

$$i^{12} = i^4 \cdot i^4 \cdot i^4 = (1)(1)(1) = 1$$

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1.4 Polynomials

poly — many

Polynomial- monomial or a sum of monomials (binomial, trinomial)

monomial → 1 term, sum → +, binomial → 2 terms, trinomial → 3 terms

Terms- The number of monomials in an expression (separated by +/−)

Like Terms- Terms that can be combined (exactly the same variables or just constants)

Ex. 1 Degree of a Polynomial

a. $\frac{1}{6}x^3y^5 - 9x^4$

deg. 8 4

8th degree

b. x^3+4x^2-6x+9

deg. 3 2 1 0

3rd degree

Ex. 2 Subtract and Simplify

a.
$(3x^2-2x+3)-(x^2+4x-2)$

$3x^2-2x+3+-x^2+-4x+2$

$(3x^2+-x^2)+(-2x+-4x)+(3+2)$

$\boxed{2x^2-6x+5}$

b. $(6-4i)-(1+3i)$

$6-4i+-1+-3i$

$(6+-1)+(-4i+-3i)$

$\boxed{5+-7i \text{ or } 5-7i}$

Ex. 3 Multiply and Simplify

a.
$2x(7x^2-3x+5)$

$2x(7x^2)+2x(-3x)+2x(5)$

$\boxed{14x^3-6x^2+10x}$

b. $4i(5-6i+6i^2)$

$4i(5)+(4i)(-6i)+4i(6i^2)$

$20i-24i^2+24i^3$

$20i-24(-1)+24(-i)$

$20i+24-24i$

$\boxed{24+-4i \text{ or } 24-4i}$

Ex. 4 Multiply Two Binomials

a.
$(3y+2)(5y+4)$

$3y(5y)+3y(4)+2(5y)+2(4)$

$15y^2+12y+10y+8$

$\boxed{15y^2+22y+8}$

b. $(4+2i)(6+9i)$

$4(6)+4(9i)+2i(6)+2i(9i)$

$24+36i+12i+18i^2$ 18(−1)

$24+48i-18 = \boxed{6+48i}$

c. Word Problem

In an AC circuit, $\boxed{\text{the voltage E}}$, $\boxed{\text{current I,}}$ and $\boxed{\text{impedance Z}}$, are related by the formula $E = I \bullet Z$. Find the voltage in a circuit with current $\underbrace{1 + 3j}_{I}$ amps, and impedance $\underbrace{7-5j}_{Z}$ ohms.

$E = I \cdot Z$

$E = (1+3j)(7-5j)$

$E = 1(7) + 1(-5j) + 3j(7) + 3j(-5j)$

$E = 7 - 5j + 21j - 15j^2 = 7 + 16j - 15j^2$

Ex. 5 Multiply Polynomials

$(n^2 + 6n - 2)(n+4)$

$n^2(n) + n^2(4) + 6n(n) + 6n(4) + -2(n) + -2(4)$

$n^3 + 4n^2 + 6n^2 + 24n - 2n + -8$

$\boxed{n^3 + 10n^2 + 22n - 8}$

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1.5 Operations on Functions

Operation	Definition
Sum Add	$(f+g)(x) = f(x) + g(x)$
Difference subtract	$(f-g)(x) = f(x) - g(x)$
Product multiply	$(f\bullet g)(x) = f(x) \bullet g(x)$
Quotient divide	$(f/g)(x) = f(x)/g(x)$

Ex. 1 Add and Subtract Functions

Given: $f(x) = 3x+1$ & $g(x) = 4x+5$

a. $(f+g)(x) = f(x)+g(x)$

$(3x+1)+(4x+5)$

$7x+6$

b. $(f-g)(x) = f(x)-g(x)$

$(3x+1)-(4x+5)$

$3x+1-4x-5$

$-1x-4$

c. $(f\cdot g)(x) = f(x)\cdot g(x)$

$(3x+1)(4x+5)$

$3x(4x)+3x(5)+1(4x)+1(5)$

$12x^2+15x+4x+5$

$12x^2+19x+5$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$\frac{3x+1}{4x+5}$ but $x \neq -\frac{5}{4}$

$4x+5 \neq 0$

$-5 \quad -5$

$\frac{4x}{4} \neq \frac{-5}{4}$

$x \neq -\frac{5}{4}$

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fog f of g of x

Composition of functions- $f \circ g$, where the range of g is a subset of the domain of $[f \circ g](x) = f[g(x)]$

Ex. 3 Evaluate Composition of Relations

$f(x) = 4x-2$
$f(1) = 4(1)-2$
$= 4-2$
$f(1) = 2$

$g(x) = 2x$
$g(-7) = 2(-7)$
$g(-7) = -14$

$g(100) = 2(100) = 200$

$f(g(x)) = 4(2x)-2$
$f(g(x)) = 8x-2$

$g(f(x)) = 2(4x-2)$
$g(f(x)) = 8x-4$

Ex. 4 Simplify Composition of Functions

Given: $f(x) = x+3$
$g(x) = x^2+x-1$

a.
$f(g(x)) = (x^2+x-1)+3$
$= x^2+x-1+3$
$\boxed{f(g(x)) = x^2+x+2}$
$g(f(x)) = (x+3)^2+(x+3)-1$
$g(f(x)) = (x+3)(x+3)+x+3-1$
$= x^2+3x+3x+9+x+3-1$
$g(f(x)) = x^2+7x+8$

Given: $f(x) = 3x^2$
$g(x) = x-1$

b.
$f(g(x)) = 3(x-1)^2$
$= 3[(x-1)(x-1)]$
$= 3[x^2-x-x+1]$
$= 3[x^2-2x+1]$
$\boxed{f(g(x)) = 3x^2-6x+3}$
$g(f(x)) = (3x^2)-1$
$g(f(x)) = 3x^2-1$

Ex. 5 Apply Composition of Functions

Tyrone Davis has $180 deducted from every paycheck for retirement. He can have the deductions taken before taxes, which reduces his taxable income. The federal income rate is 18%. If Tyrone earns $2200 every pay period, find the difference in his net income if he has the retirement deduction taken before or after taxes.

before
2200 − 180 = 2020
2020 · .82 = $1656.40
take home net income

after
2200 × 82%
2200 × .82 = 1804
1804 − 180 = $1624
take home

1656.40 − 1624 = $32.40

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$\sqrt[2]{4}$ = square root

1.6 Roots of Real Numbers

(positive)

$\sqrt[a]{b}$

Principal Root- the nonnegative root when there is more than one real root

b+   b-

A	$\sqrt[a]{b}$ If b >0	$\sqrt[a]{b}$ If b <0	B = 0
Even root	+ One positive root, − one negative root	No real roots 2 imaginary roots	One real root, 0
Odd root	One positive root, no negative roots	No positive roots, one negative root	0

Ex. 1 Find Roots

a. $\sqrt{36x^4}$

6 6

$\sqrt{6\cdot 6\cdot xx\,xx}$

$= 6\cdot x\cdot x$

$= 6x^2$

b. $-\sqrt{(y^2+2)^8}$

$-\sqrt{(y^2+2)(y^2+2)(y^2+2)(y^2+2)(y^2+2)(y^2+2)(y^2+2)(y^2+2)}$

$-(y^2+2)^4$

c. $\sqrt[5]{32x^{15}y^{20}}$

8 4, 4 2, 2 2, 2 2

$= \sqrt[5]{2\cdot 2\cdot 2\cdot 2\cdot 2\,xxxxx\,xxxxx\,xxxxx\,yyyy\,yyyy\,yyyy\,yyyy\,yyyy}$

$= 2|x^3|y^4$

d.

Ex. 2 Simplify Using Absolute Value

a. $\sqrt[8]{x^8}$

$\sqrt[8]{x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot xx}$

$|x|$

b. $\sqrt[4]{81(a+1)^{12}}$

9 9, 3 3 3 3

$\sqrt[4]{3\cdot 3\cdot 3\cdot 3\,(a+1)(a+1)(a+1)(a+1)\,(a+1)(a+1)(a+1)(a+1)\,(a+1)(a+1)(a+1)(a+1)}$

$= 3|(a+1)^3|$

Ex. 3 Find the roots of Negative Numbers

a. $\sqrt[3]{-8x^6y^{13}}$

4 2, 2 2

$= -\sqrt[3]{(2\cdot 2\cdot 2)\cdot xxx\,xxx\,yyy\,yyy\,yyy\,yyy\cdot y}$

$-2x^2y^4\sqrt[3]{y}$

b.

$\sqrt{-49x^2}$

7 7

$\sqrt{-1\cdot 7\cdot 7\,xx}$

$7\cdot x\cdot i = 7|x|\cdot i$

$\sqrt{4} = \sqrt{2 \cdot 2} = \pm 2 \qquad \sqrt{4} = \sqrt{(-2)(-2)} = -2$

2 2

$\sqrt{50} = \sqrt{5 \cdot 5 \cdot 2} = 5\sqrt{2}$

25 2

5 5

or $\sqrt{5^2 \cdot 2} = \sqrt{5^2} \cdot \sqrt{2}$

$= 5\sqrt{2}$

$\sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2$

4 2

2 2

$\sqrt[3]{12} = \sqrt[3]{2 \cdot 2 \cdot 3} = \sqrt[3]{12}$

6 2

2 3

$\sqrt[3]{-64} = -\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = -2 \cdot 2 = -4$

8 8

2 4 2 4

2 2 2 2

$\sqrt{48x^2y^6z^2} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z}$

6 8

3 2 4 2

2 2

$2 \cdot 2 \cdot x \cdot y \cdot y \cdot y \cdot z\sqrt{3}$

$4|xy^3z|\sqrt{3}$

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1.7 Radical Expressions

Ex. 1 Square Root of a Product

a. $\sqrt{16p^8q^7} = 2\cdot 2pppp\,qqq\sqrt{q}$

$4p^4|q^3|\sqrt{q}$

2 8, 4 2, 2 2

$\sqrt{2\cdot2\cdot2\cdot2\cdot pppppppp\,qqqqqqq}$

b. $\sqrt{32a^2b^9}$

8 4, 4 2

$\sqrt{4\cdot4\cdot2\,aa\,bbbbbbbbb}$

$4|a|b^4\sqrt{2b}$

c. $\sqrt{\frac{9x^4}{25y^2}} = \sqrt{\frac{3\cdot3\,xx\,xx}{5\cdot5\,yy}}$

$\frac{3x^2}{5|y|}$

d. $\sqrt[3]{\frac{5}{4a}} = \frac{\sqrt[3]{5}}{\sqrt[3]{4a}}\cdot\frac{\sqrt[3]{4a\cdot4a}}{\sqrt[3]{4a\cdot4a}}$

$= \frac{\sqrt[3]{5\,2\cdot2\cdot2\,2aa}}{4a} = \frac{2\sqrt[3]{10a^2}}{4|a|}$

$= \frac{\sqrt[3]{10a^2}}{2|a|}$

Ex. 2 Simplifying Quotients

a. $\sqrt{\frac{x^4}{y^5}} = \frac{\sqrt{xxxx}}{\sqrt{yyyyy}}$

$\frac{x\cdot x}{y\cdot y\sqrt{y}}\cdot\frac{\sqrt{y}}{\sqrt{y}} = \frac{x^2\sqrt{y}}{y^2\sqrt{y\cdot y}}$

$\boxed{\frac{x^2\sqrt{y}}{|y^3|}}$

b. $\sqrt{\frac{18x^3}{36y^2}} = \frac{\sqrt{xx}\,x}{\sqrt{2}\,yy}$

$= \frac{x\sqrt{x}}{y\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{|x|\sqrt{2x}}{2|y|}$

Ex. 3 Multiply Radicals

a. $(-2\sqrt{15})(4\sqrt{21})$

$= (-2)(4)\sqrt{15\cdot21}$

3 5 3 7

$-8\sqrt{3\cdot3\cdot5\cdot7} = -8\cdot3\sqrt{5\cdot7} = \boxed{-24\sqrt{35}}$

b. $6\sqrt[3]{9n^2}\cdot3\sqrt[3]{24n}$

$(6)(3)\sqrt[3]{9n^2\cdot24n}$

3 3, 8 3, 4 2, 2 2

$18\sqrt[3]{3\cdot3\cdot3\,2\cdot2\cdot2\,n\cdot n\cdot n}$

$18\cdot3\cdot2\cdot n = \boxed{108|n|}$

c. $\sqrt{-10}\,\sqrt{-15}$

-1 10, 2 5; -1 15, 3 5

$\sqrt{-1\cdot-1\,2\cdot3\,5\cdot5} = -1\cdot5\sqrt{2\cdot3}$

$= \boxed{-5\sqrt{6}}$

d. $\sqrt{-125x^5}\,\sqrt{5x}$

-1 125, 5 25, 5 5

$= \sqrt{-1\,5\cdot5\,5\cdot5\,x\cdot x\,xxxx}$

$5\cdot5 i|x^3|$ $25x^3 i$

$2\sqrt{2} + 3\sqrt{2} - 2\sqrt{3} = \boxed{5\sqrt{2} - 2\sqrt{3}}$

Ex. 4 Add and Subtract Radicals

a. $2\sqrt{12} - 3\sqrt{27} + 2\sqrt{48}$

$2\sqrt{2\cdot2\cdot3} - 3\sqrt{3\cdot3\cdot3} + 2\sqrt{2\cdot2\cdot2\cdot2\cdot3}$

$2\cdot2\sqrt{3} - 3\cdot3\sqrt{3} + 2\cdot2\cdot2\sqrt{3}$

$4\sqrt{3} - 9\sqrt{3} + 8\sqrt{3}$

$-5\sqrt{3} + 8\sqrt{3}$

$3\sqrt{3}$

b. $\sqrt{-28} + \sqrt{-63}$

$\sqrt{(-1)(2\cdot2)7} + \sqrt{(-1)(3\cdot3)7}$

$2i\sqrt{7} + 3i\sqrt{7} =$

$\boxed{5i\sqrt{7}}$

Ex. 5 more Multiply Radicals

a. $(3\sqrt{5} - 2\sqrt{3})(2 + \sqrt{3})$

$(3\sqrt{5})(2) + (3\sqrt{5})(\sqrt{3}) + (-2\sqrt{3})(2) + (-2\sqrt{3})(\sqrt{3})$

$6\sqrt{5} + 3\sqrt{15} - 4\sqrt{3} - 2\sqrt{3\cdot3}$

$6\sqrt{5} + 3\sqrt{15} - 4\sqrt{3} - 2(3)$

$\boxed{6\sqrt{5} + 3\sqrt{15} - 4\sqrt{3} - 6}$

b. $(5\sqrt{3} - 6)(5\sqrt{3} + 6)$

$(5\sqrt{3})(5\sqrt{3}) + (5\sqrt{3})(6) + (-6)(5\sqrt{3}) + (-6)(6)$

$25\sqrt{3\cdot3} + 30\sqrt{3} - 30\sqrt{3} - 36$

$25\cdot3 - 36 = 75 - 36 = \boxed{39}$

c. $(1+3i)(7-5i)$

$(1)(7) + (1)(-5i) + (3i)(7) + (3i)(-5i)$

$7 - 5i + 21i - 15i^2$

$7 + 16i - 15(-1) = 7 + 16i + 15 = 22 + 16i$

d.

Ex. 6 Use a Conjugate to Rationalize a Denominator

a.

b.

b.

d.

8/23 1.7 notes continued

conjugate - binomial
- 1st term is the same
- 2nd term is the same
- sign (+/-) of the 2nd term is opposite

2 of them

$a+\sqrt{b} \longleftrightarrow a-\sqrt{b}$

$3-\sqrt{5} \longleftrightarrow 3+\sqrt{5}$

$2-3i \longleftrightarrow 2+3i$

Product of Conjugates

$(a+\sqrt{b})(a-\sqrt{b}) = a(a) + a(-\sqrt{b}) + (\sqrt{b})(a) + (\sqrt{b})(-\sqrt{b})$

$a^2 -a\sqrt{b} + a\sqrt{b} - \sqrt{b \cdot b}$

$\boxed{a^2 - b}$

$(3-\sqrt{5})(3+\sqrt{5}) = (3)(3) + (3)(\sqrt{5}) + (-\sqrt{5})(3) + (-\sqrt{5})(\sqrt{5})$

$9 + 3\sqrt{5} - 3\sqrt{5} - \sqrt{5 \cdot 5}$

$9 - 5 = \boxed{4}$

$(2-3i)(2+3i) = 2(2) + 2(3i) + (-3i)(2) + (-3i)(3i)$

$4 + 6i - 6i - 9i^2$

$4 - 9(-1) = 4+9 = \boxed{13}$

★ you cannot divide by a $\sqrt{\ }$ or an i

★ to change that, multiply by the conjugate of the denominator

Ex. Use a conjugate to Rationalize a denominator

a) $$\frac{4}{4+\sqrt{3}} \cdot \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{4(4)+4(-\sqrt{3})}{4(4)+4(-\sqrt{3})+(\sqrt{3})(4)+(\sqrt{3})(-\sqrt{3})}$$

FFOO

$$= \frac{16-4\sqrt{3}}{16-4\sqrt{3}+4\sqrt{3}-\sqrt{3\cdot 3}}$$

$$= \frac{16-4\sqrt{3}}{16-3}$$

$$= \frac{16-4\sqrt{3}}{13} = \frac{16}{13}-\frac{4}{13}\sqrt{3}$$

b) $$\frac{2\sqrt{3}+6}{9-\sqrt{7}} \cdot \frac{9+\sqrt{7}}{9+\sqrt{7}}$$

FFOO

$$= \frac{(2\sqrt{3})(9)+(2\sqrt{3})(\sqrt{7})+6(9)+6(\sqrt{7})}{9(9)+9(\sqrt{7})+(-\sqrt{7})(9)+(-\sqrt{7})(\sqrt{7})}$$

$$= \frac{18\sqrt{3}+2\sqrt{21}+54+6\sqrt{7}}{81+9\sqrt{7}-9\sqrt{7}-\sqrt{7\cdot 7}}$$

$$= \frac{18\sqrt{3}+2\sqrt{21}+54+6\sqrt{7}}{81-7}$$

$$= \frac{18\sqrt{3}+2\sqrt{21}+54+6\sqrt{7}}{74} \text{ or } \frac{18}{74}\sqrt{3}+\frac{2}{74}\sqrt{21}+\frac{54}{74}+\frac{6}{74}\sqrt{7}$$

1.7 continued.

c. $$\frac{3i}{2+4i}\cdot\frac{2-4i}{2-4i} = \frac{(3i)(2)+(3i)(-4i)}{(2)(2)+(2)(-4i)+(4i)(2)+(4i)(-4i)}$$

FFOO

$$= \frac{6i-12i^2}{4-8i+8i-16i^2}$$

$$= \frac{6i-12(-1)}{4-16(-1)}$$

$$= \frac{6i+12}{4+16} = \frac{12+6i}{20}$$

$$= \frac{12^{\div 4}}{20_{\div 4}} + \frac{6^{\div 2}}{20_{\div 2}}i = \boxed{\frac{3}{5}+\frac{3}{10}i}$$

d. $$\frac{5+i}{2i-3}\cdot\frac{2i+3}{2i+3}$$

FFOO

$$= \frac{5(2i)+5(3)+(i)(2i)+i(3)}{2i(2i)+2i(3)+(-3)(2i)+(-3)(3)}$$

$$= \frac{10i+15+2i^2+3i}{4i^2+6i-6i-9}$$

$$= \frac{13i+15+2(-1)}{4(-1)-9} = \frac{13i+15-2}{-4-9}$$

$$= \frac{13+13i}{-13} = \frac{13}{-13}+\frac{13}{-13}i = -1-i$$

1.7 continued.

c. $$\frac{3i}{2+4i} \cdot \frac{2-4i}{2-4i} = \frac{(3i)(2)+(3i)(-4i)}{(2)(2)+(2)(-4i)+(4i)(2)+(4i)(-4i)}$$

FFOO

$$= \frac{6i - 12i^2}{4 - 8i + 8i - 16i^2}$$

$$= \frac{6i - 12(-1)}{4 - 16(-1)}$$

$$= \frac{6i + 12}{4 + 16} = \frac{12 + 6i}{20}$$

$$= \frac{12^{\div 4}}{20_{\div 4}} + \frac{6^{\div 2}}{20_{\div 2}} i = \boxed{\frac{3}{5} + \frac{3}{10} i}$$

d. $$\frac{5+i}{2i-3} \cdot \frac{2i+3}{2i+3}$$

FFOO

$$= \frac{5(2i) + 5(3) + (i)(2i) + i(3)}{2i(2i) + 2i(3) + (-3)(2i) + (-3)(3)}$$

$$= \frac{10i + 15 + 2i^2 + 3i}{4i^2 + 6i - 6i - 9}$$

$$= \frac{13i + 15 + 2(-1)}{4(-1) - 9} = \frac{13i + 15 - 2}{-4 - 9}$$

$$= \frac{13 + 13i}{-13} = \frac{13}{-13} + \frac{13}{-13} i = -1 - i$$