

Ex. 4 Transformation of a Quadratic Function

graph using transformations

$$y = (x+3)^2 - 4 \text{ purple}$$

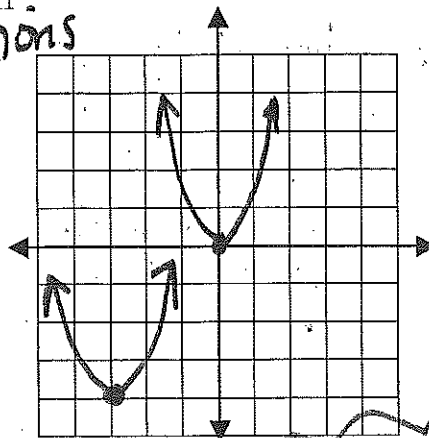
$$V(-3, -4)$$

parent graph - green

$$y = x^2$$

$$V(0, 0)$$

left 3
down 4



Classwork:

p325 #5, 6, 15, 20

3.1.1 Polynomial Functions

Polynomial in one variable - a polynomial that has one type of variable in it (all x's)

Degree of a Polynomial - the greatest exponent of its variable

L.C. Leading Coefficient - the coefficient of the term with the highest degree

Polynomial Function - polynomial equation used to represent a function

Ex. 1 Find Degree and Leading Coefficients

a. $7x^4 + 5x^2 + x - 9$
4th; L.C. = 7

b. $8x^2 + 3(x^4) - 2(y^2)$ NO

c. $7x^6 - 4x^3 + \left(\frac{1}{x}\right)$ NO

d. $\frac{1}{2}x^2 + 2x^3 - x^5$
5th degree; L.C. = -1

Ex. 2 Evaluate a Polynomial Function

a. $f(r) = 3r^2 - 3r + 1$ $r = 1, 2, 3$
 $f(1) = 3(1)^2 - 3(1) + 1 = 3(1) - 3(1) + 1 = 3 - 3 + 1 = 1$ $f(1) = 1$
 $f(2) = 3(2)^2 - 3(2) + 1 = 3(4) - 3(2) + 1 = 12 - 6 + 1 = 7$ $f(2) = 7$
 $f(3) = 3(3)^2 - 3(3) + 1 = 3(9) - 3(3) + 1 = 27 - 9 + 1 = 19$ $f(3) = 19$

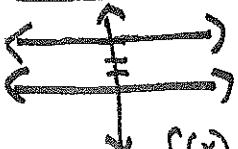





Ex. 3 Functional Values of Variables

$$f(x) = x^3 + 4x^2 - 5x$$

a. find $f(a^2)$
 $f(a^2) = (a^2)^3 + 4(a^2)^2 - 5(a^2)$
 $a^2 \cdot a^2 \cdot a^2 + 4a^2 \cdot a^2 - 5a^2$
 $f(a^2) = a^6 + 4a^4 - 5a^2$

b. $g(x) = x^2 + 3x + 4$
 find: $g(a+1) - 2g(a)$
 $(a+1)^2 + 3(a+1) + 4 - 2(a^2 + 3a + 4)$
 $(a+1)(a+1) + 3a + 3 + 4 - 2a^2 - 6a - 8$
 $a^2 + 1a + 1a + 1 + 3a + 3 + 4 - 2a^2 - 6a - 8$
 $-1a^2 - 1a$ or $-a^2 - a$

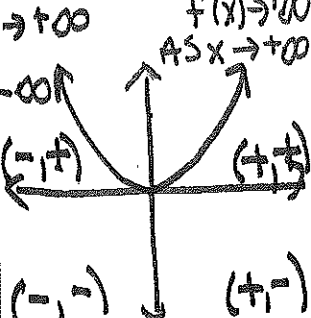
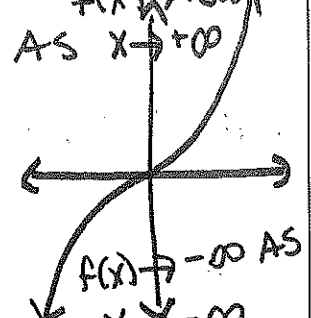
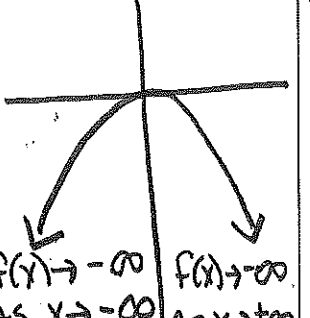
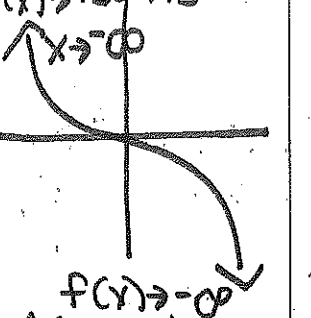
Graphs of Polynomial Functions

<u>Constant function</u>  $f(x) = 3$ 0 degree	<u>Linear function</u>  $f(x) = x$ 1 st degree	<u>Quadratic function</u>  $f(x) = x^2$ 2 nd degree
<u>Cubic function</u>  $f(x) = x^3$ 3 rd degree	<u>Quartic function</u>  $f(x) = x^4$ 4 th degree	<u>Quintic function</u>  $f(x) = x^5$ 5 th degree

★ Change in directions or # bumps or # turns + 1 = degree

End behavior - behavior of the graph as x approaches positive or negative infinity.
or most # turns = degree - 1

End Behavior of Polynomial Functions

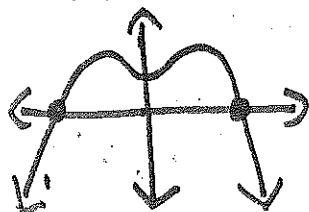
Degree: Even L.C.: + End behavior:  $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$ $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$ $(-1, +)$ $(+1, +)$ $(-1, -)$ $(+1, -)$	Degree: odd L.C.: + End behavior:  $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$ $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$	Degree: even L.C.: - End behavior:  $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$ $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$	Degree: odd L.C.: - End behavior:  $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$
--	--	--	--

Ex. 4 Graphs of Polynomial Functions

For each graph,

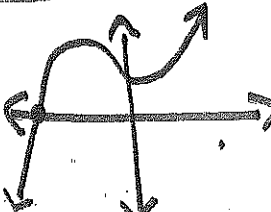
- Describe the end behavior
- Determine whether it is an odd-degree or even-degree polynomial function
- State the number of real zeros \rightarrow x intercepts (crosses x axis)

a.



- $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$
 $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$
- EVEN
- 3 real zeros

b.



- $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$
 $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$
- ODD
- 1 real zero

c.



- $f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$
 $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$
- EVEN
- 2 real zeros

turn plots off $\rightarrow 2nd y=$, #4, Enter
(PLOT)

3.12 Graphing Polynomial Functions

Location Principle - way for locating zeros (x-intercepts)

Relative Maximum - when no other nearby points have a greater y-coordinate

(highest point within an area (turning point))

Relative Minimum - when no other nearby points have a lesser y-coordinate

(lowest point within an area (turning point))

max # Turning points - the degree of the problem - 1 (how many maxs and mins)

Ex. 1 Graph a Polynomial Function

(table)

X	y
-5	30
-4	5
-3	-4
-2	-3
-1	2
0	5
1	0
2	-19
3	-58
4	-123
5	-220

$$f(x) = -x^3 - 4x^2 + 5$$

$$\text{deg} = 3$$

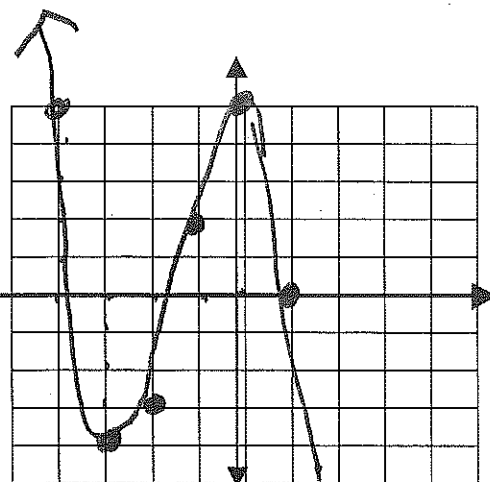
$$\text{L.C.} = -1$$

$$\text{max \# turning pts} = 2$$

$$\text{Zeros: } (-1.3, 0) (0.7, 0) (1.0, 0)$$

$$\text{max: } (0, 5)$$

$$\text{min: } (2.8, -44)$$



Ex. 2 Locate Zeros of a Function

$$f(x) = x^4 - x^3 - 4x^2 + 1$$

$$\text{deg: } 4$$

$$\text{L.C.} = +1$$

$$\text{max \# turning pts} = 3$$

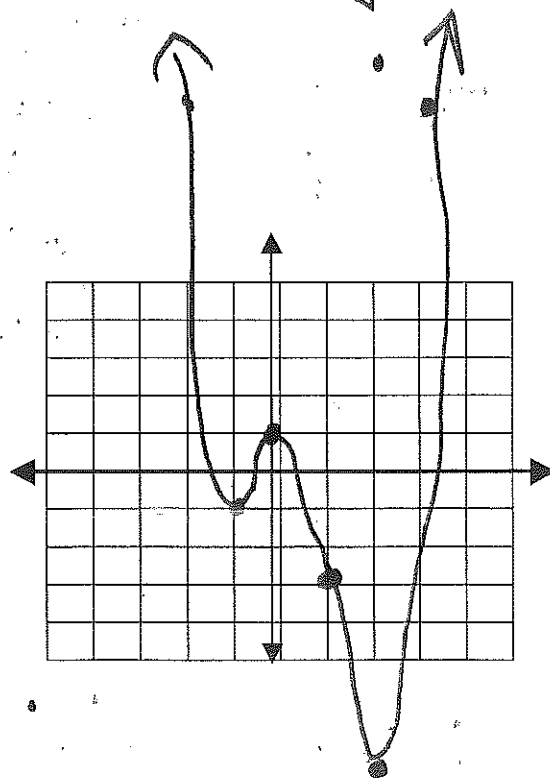
$$\text{Zeros: } (-1.5, 0) (-1.3, 0)$$

$$(1.5, 0) (2.4, 0)$$

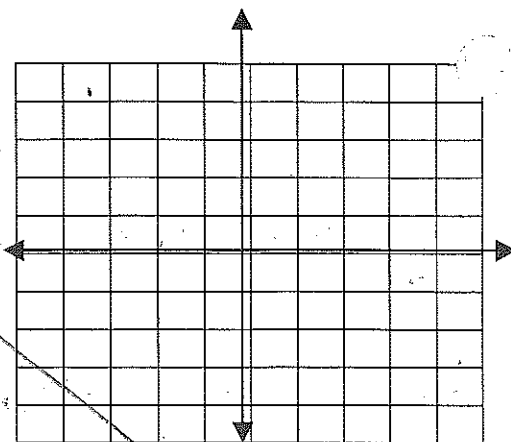
$$\text{max: } (0, 1)$$

$$\text{min: } (-1, -1) (1.9, -7.2)$$

X	y
-4	257
-3	73
-2	9
-1	-1
0	1
1	-3
2	-7
3	19
4	129

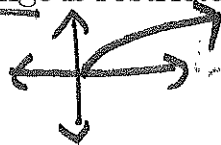


Ex. 3 Find the equation of the polynomial function given the graph



3.13 Square Root Functions and Inequalities

Square root function - a function that contains a square root of a variable. The inverse of a quadratic is a square root function if the range is restricted to nonnegative numbers. (otherwise the graph is not a function)



Ex. 1 Graph a Square Root Function

X	Y
$-\frac{4}{3}$	0
0	2
4	4

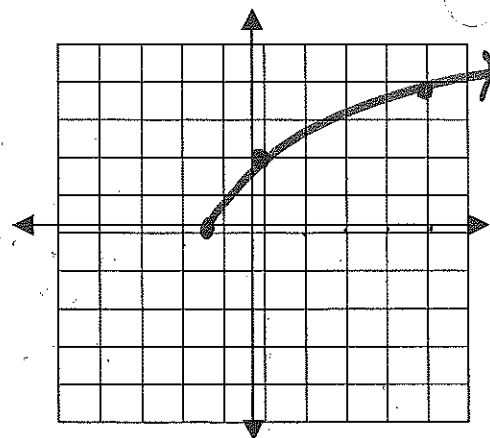
$$y = \sqrt{3x+4}$$

$$3x+4 \geq 0$$

$$\begin{array}{r} -4 \quad -4 \\ 3x \geq -4 \\ \frac{3x}{3} \geq \frac{-4}{3} \\ x \geq -\frac{4}{3} \end{array}$$

$$D: \left[-\frac{4}{3}, \infty\right)$$

$$R: [0, \infty)$$



Ex. 2 Solve a Square Root Problem

X	Y
0	$\frac{2}{3}$
6	$\frac{5}{3}$

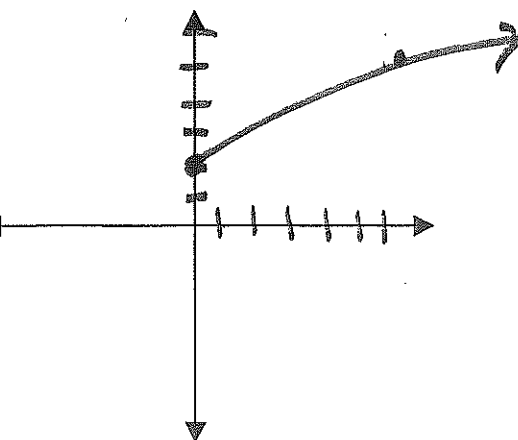
$$y = \sqrt{\frac{3}{2}x} + 2$$

$$\left(\frac{2}{3}\right) \frac{3}{2}x \geq 0 \left(\frac{2}{3}\right)$$

$$x \geq 0$$

$$D: [0, \infty)$$

$$R: [2, \infty)$$



Ex. 3 Graph a Square Root Inequality

$$\sqrt{7(\frac{2}{7})} - 2 = \sqrt{2-2} = 0$$

x	y
$\frac{2}{7}$	0
3	4.4

$$y = \sqrt{7x-2}$$

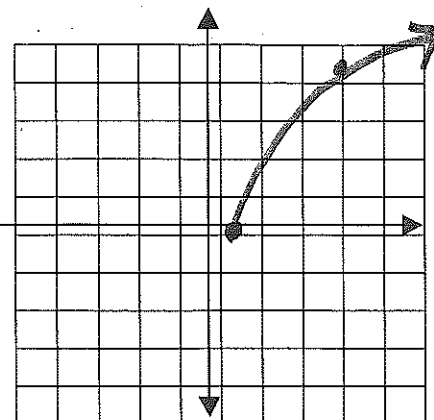
$$7x-2 \geq 0$$

$$\begin{array}{r} +2 \quad +2 \\ \hline 7x \geq 2 \\ \frac{7x}{7} \geq \frac{2}{7} \end{array}$$

$$x \geq \frac{2}{7}$$

$$D: [\frac{2}{7}, \infty)$$

$$R: [0, \infty)$$



3.14 Inverse Functions and Relations

Inverse Relation - the set of ordered pairs obtained by reversing the coordinates of each original ordered pair. Domain and Range switch positions

Inverse Function - $f^{-1}(x)$

Identity Function - $I(x) = x$

$$f(x) = \{(1,2), (3,4), (5,6)\}$$

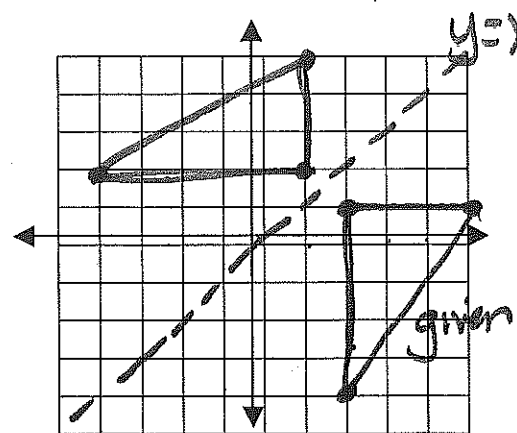
$$f^{-1}(x) = \{(2,1), (4,3), (6,5)\}$$

Ex. 1 Find an Inverse Relation

given: $(2,1), (5,1), (-2,-4)$ Δ

inverse: $(1,2), (1,5), (-4,-2)$

inverses are reflected over the line $y = x$



Ex. 2 Find an Inverse Function

Steps

① replace $f(x)$ with y

② SWITCH x & y

③ solve for y

④ replace y with $f^{-1}(x)$

work

$$f(x) = \frac{x+6}{2} = \frac{1}{2}x + 3$$

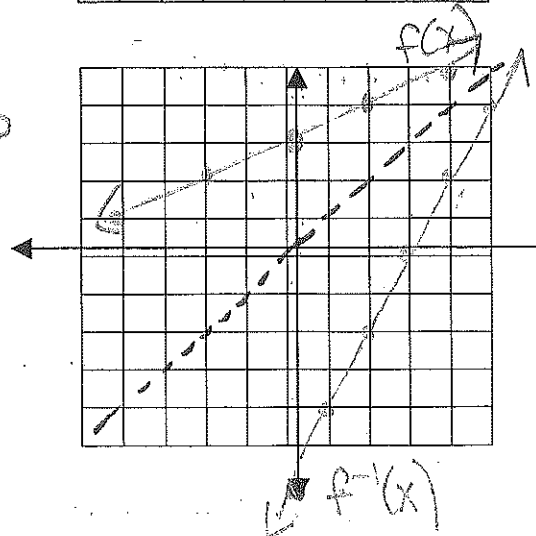
$$y = \frac{x+6}{2}$$

$$x = \frac{y+6}{2}$$

$$2x = y+6$$

$$2x-6 = y$$

$$f^{-1}(x) = 2x-6$$



Ex. 3 Verify Two Functions are Inverses

$$f(x) = 5x + 10$$

yes they are
inverses

$$g(x) = \frac{1}{5}x - 2$$

$$y = \frac{1}{5}x - 2$$

$$x = \frac{1}{5}y - 2$$

$$5(x+2) = \frac{1}{5}y \left(\frac{5}{1}\right)$$

$$5x + 10 = y$$

$$g^{-1}(x) = 5x + 10$$

3.15 Absolute Value Functions

Absolute Value Function - in the form $f(x) = |x|$

distance from zero \rightarrow always +

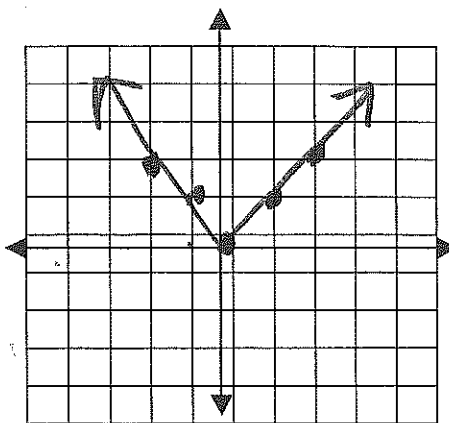
Ex. 1

$$f(x) = |x|$$

x	y
-2	2
-1	1
0	0
1	1
2	2

$$D: (-\infty, \infty)$$

$$R: [0, \infty)$$



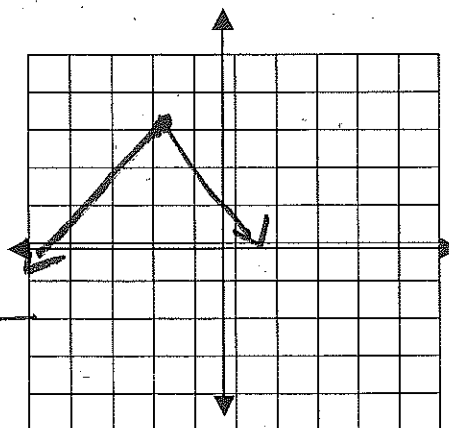
Ex. 2 Transformation of Absolute Value Functions

$$f(x) = -|x+2|+3$$

left 2

up 3

refl. over x axis



x intercept $y=0$ set numerator = 0

3.16 Graphing Rational Functions

Rational Function - $f(x) = p(x)/q(x)$ where $q(x) \neq 0$

never = by zero

fraction w/ x in denominator

Continuity - graphs that can be drawn without having breaks.

Asymptote - breaks in continuity (lines); found by solving for x in the denominator

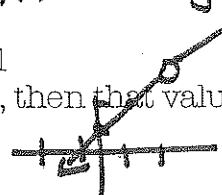
line the graph approaches as $x \rightarrow \infty$ or $y \rightarrow \infty$

Point discontinuity - hole in a graph (point) when a rational

expression can be reduced and the denominator becomes true, then that value is a hole

$$f(x) = \frac{(x+1)(x-2)}{(x-2)} \approx x+1 \quad x-2 \neq 0$$

$$x \neq 2$$



- point discontinuity - set denominator = 0 & solve after you reduce fraction

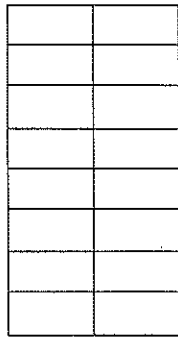
Ex. 1 Vertical Asymptotes and Point Discontinuity

V.A. • vertical asymptote - vertical line graph approaches graph $\rightarrow \pm \infty$ * Set denominator = 0

- Horizontal asymptote - horizontal line graph approaches as $x \rightarrow \pm \infty$
 - if degree on top > degree on bottom NO H.A.
 - if degree on top < degree on bottom $y = 0$
 - if degree on top = degree on bottom

Ex. 2 Graph with a Vertical Asymptote

Given:



$$f(x) = \frac{-4}{x^2-3x}$$

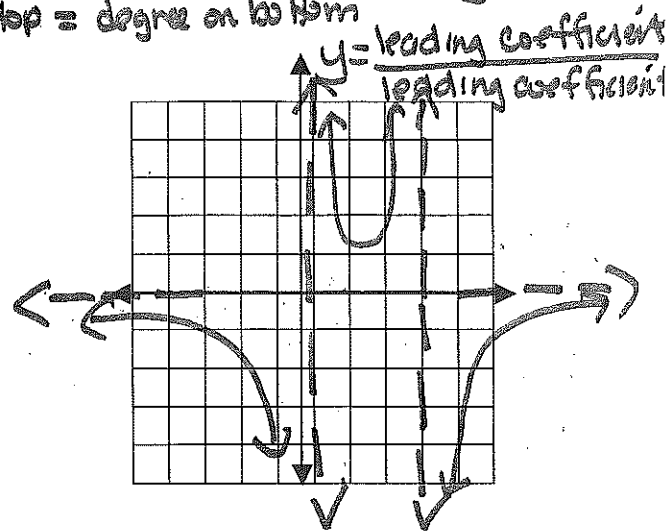
discontinuity:
 $x^2-3x=0$

$$x(x-3)=0$$

$$V.A. \quad x=0 \quad x-3=0$$

$$x=3$$

$$H.A. \quad y=0$$



Ex. 3 Graph with Point Discontinuity

$$f(x) = \frac{x^2+x-6}{-4x^2-16x-12} = \frac{(x-2)(x+3)}{-4(x+3)(x+1)}$$

$$= \frac{(x-2)}{-4(x+1)}$$

Point discontinuity (hole)

$$x+3=0$$

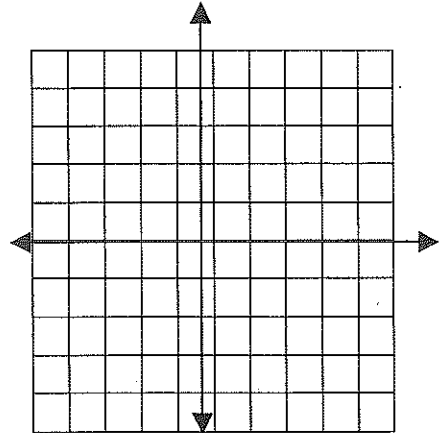
$$x=-3$$

$$V.A. \quad -4(x+1)=0$$

$$x+1=0$$

$$V.A. \quad x=-1$$

$$H.A. \quad y = -\frac{1}{4}$$



3.17 Exponential Functions

Exponential Function - $y = ab^x$

Has the following characteristics

- The function is continuous and one-to-one.
- The domain = all real numbers
- X-axis is an asymptote $y = 0$
- Range = positive numbers if $a > 0$ and all negative numbers if $a < 0$
- Y-intercept is $(0, a)$
- $Y = ab^x$ and $y = a(1/b)^x$ are reflections across the y-axis

has x in the exponent

D: $(-\infty, \infty)$
 R: $(0, \infty)$

$$y = a(b)^x$$

Exponential growth - $b > 1$

Exponential decay - $0 < b < 1$ (fraction less than 1)

Exponential Equations - are equations in which variables occur as exponents

Exponential Inequalities - Same application as Exponential Equations

Ex. 1 Graph an Exponential Function

x	y
-1	2.5
0	5
1	10
2	20
3	40

$$y = 5 \cdot 2^x$$

$$5(2)^{-1} = \frac{5}{2} = 2.5$$

$$5(2)^0 = 5 \cdot 1 = 5$$

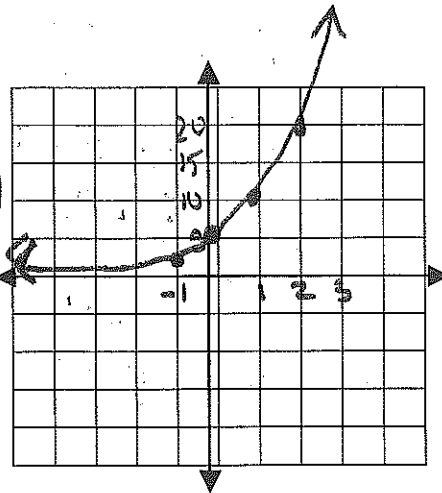
$$5(2)^1 = 5 \cdot 2 = 10$$

$$5(2)^2 = 5 \cdot 4 = 20$$

$$5(2)^3 = 5 \cdot 8 = 40$$

$$D: (-\infty, \infty)$$

$$R: (0, \infty)$$



Ex. 2 Exponential Growth and Decay

a. $y = \left(\frac{1}{5}\right)^x$

$b = \frac{1}{5}$ decay

b. $y = 3 \cdot 4^x$

$b = 4$ growth

c. $y = 1.2^x$

$b = 1.2$ growth

Ex. 3 Simplify Expressions with Rational Exponents

a. $2^{\frac{1}{3}} \cdot 2^{\frac{2}{3}}$

$$= 2^{\frac{1}{3} + \frac{2}{3}} = 2^1 = 2$$

b. $(7^{\frac{1}{2}})^3$

$$= 7^{(\frac{1}{2} \cdot 3)} = 7^{\frac{3}{2}} = 7\sqrt{7}$$

Ex. 4 Solve Exponential Equations

a. $3^{2n+1} = 81$

$$3^{2n+1} = 3^4$$

$$2n+1 = 4$$

$$2n = 3$$

$$n = \frac{3}{2}$$

b. $4^{2x} = 8^{x-1}$

$$(2^2)^{2x} = (2^3)^{x-1}$$

$$2^{4x} = 2^{3x-3}$$

$$4x = 3x - 3$$

$$x = -3$$

Ex. 5 Solve Exponential Inequalities

a. $4^{3p-1} > \frac{1}{256} = \frac{1}{4^4}$

$$4^{3p-1} > 4^{-4}$$

$$3p-1 > -4$$

$$3p > -3$$

$$p > -1$$

b. 10^{p-1}

log is the inverse of an exponential

3.18 Logarithms and Logarithmic Functions $y = \log_b x \Leftrightarrow y = 10^x$

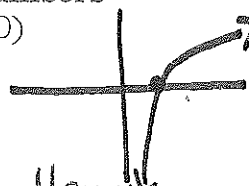
Logarithm - $y = \log_b x$, or is the inverse of $y = b^x$ ($x = b^y$),
where y is the logarithm of x . Read as $y = \log$ base b of x .

Logarithm with base b - if b and $x > 0$ and $b \neq 1$, $\log_b x$ is defined
as the exponent y that makes the
equation $b^y = x$ true

Logarithmic Function - $y = \log_b x$

Characteristics

1. Function is continuous one-to-one
2. Domain is all real positive numbers
3. Y-axis is an asymptote
4. Range = all real numbers
5. X-intercept is (1,0)



$\log_b X = y$ then $b^y = X$

$\log_b 1 = 0$

$\log_b 0 = \text{undefined}$

$\log_b b = 1$

$\log_b b^x = x$

$b^{\log_b x} = x$

Ex. 1 Logarithmic to Exponential Form

a. $\log_8 1 = 0$
 $8^0 = 1$

$\log_2 16 = 4$
 $2^4 = 16$

b. $\log_2 \frac{1}{16} = -4$
 $2^{-4} = \frac{1}{16}$

Ex. 2 Exponential to Logarithmic Form

a. $10^3 = 1000$
 $\log_{10} 1000 = 3$

$3^4 = 81$
 $\log_3 81 = 4$

b. $9^{\frac{1}{2}} = 3$
 $\log_9 3 = \frac{1}{2}$

Ex. 3 Evaluate Logarithmic Expressions

$\log_2 64 = 6$
 $\log_2 64 = x$
 $2^x = 64$

$2^x = 2^6$
 $x = 6$

Ex. 4 Inverse Property of Exponents and Logarithms

a. $\log_6 6^8 = 8$
 $\log_6 6^8 = x$
 $6^x = 6^8$ $x = 8$

b.

Ex. 5 Solve a Logarithmic Equation

$\log_4 n = \frac{5}{2}$
 $4^{\frac{5}{2}} = n$
 $(2^2)^{\frac{5}{2}} = n$
 $n = 2^5 = 32$

~~Ex. 6~~ Solve a Logarithmic Inequality

Ex. 7 Solve Equations with Logarithms on Each Side

$$\log_5(p^2 - 2) = \log_5 p$$

$$p^2 - 2 = p$$

$$p^2 - p - 2 = 0$$

$$(p - 2)(p + 1) = 0$$

~~Ex. 8~~ Solve Inequalities with Logarithms on Each Side

$$p - 2 = 0 \quad p + 1 = 0$$

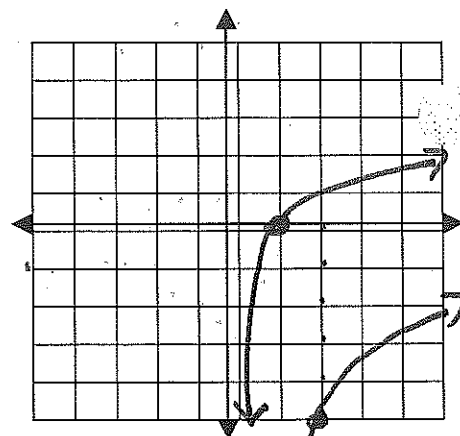
$$+2 \quad +2 \quad -1 \quad -1$$

$$p = 2 \quad p = -1$$

Ex. 9 Graph a Logarithmic Function

$$y = \log x \quad y = \log(x - 1) - 5$$

right 1
down 5



3.19 Properties of Logarithms

Type of Property	Definition	Example
<u>Product</u> \times	<u>Sum</u> of the <u>logarithms</u> of its factors	$\log_b(ac) = \log_b a + \log_b c$
Quotient \div	<u>Difference</u> of the <u>logarithms</u> of the numerator and denominator	$\log_b\left(\frac{a}{c}\right) = \log_b a - \log_b c$
<u>Power</u> <u>exponent</u>	\times <u>Product</u> of the <u>logarithm</u> and the exponent	$\log_b a^r = r \log_b a$

$$\log_3(4 \cdot 7) = \log_3 4 + \log_3 7$$

$$\log_4\left(\frac{16}{7}\right) = \log_4 16 - \log_4 7$$

$$\log_5 3^2 = 2 \log_5 3$$

Ex. 1 Use the Product Property

find $\log_2 48$

$$= \log_2 (2^4 \cdot 3) = \log_2 2^4 + \log_2 3$$

$$= 4 \log_2 2 + \log_2 3 = 4(1) + 1.5850$$

$$= 4 + 1.5850 = 5.5850$$

use $\log_2 3 \approx 1.5850$

$$\begin{array}{c} 48 \\ \wedge \\ 2 \cdot 24 \\ \wedge \\ 2 \cdot 12 \\ \wedge \\ 2 \cdot 6 \\ \wedge \\ 2 \cdot 3 \end{array}$$

Ex. 2 Use the Quotient Property

use $\log_3 5 \approx 1.4650$ & $\log_3 20 \approx 2.7268$ to find $\log_3 4$

$$\log_3 4 = \log_3 \left(\frac{20}{5} \right) = \log_3 20 - \log_3 5$$

$$= 2.7268 - 1.4650 = 1.2618$$

Ex. 3 Use the Power Property

use $\log_4 6 \approx 1.2925$ to find $\log_4 36$

$$\log_4 36 = \log_4 6^2 = 2 \log_4 6 = 2(1.2925) = 2.5850$$

Ex. 4 Solve Equations Using Properties

$$3 \log_5 x - \log_5 4 = \log_5 16$$

$$\log_5 x^3 - \log_5 4 = \log_5 16$$

$$\log_5 \left(\frac{x^3}{4} \right) = \log_5 16$$

$$\frac{x^3}{4} = 16 \quad x^3 = 64 \quad \boxed{x=4}$$

$$\log_4 x + \log_4 (x-6) = 2$$

$$\log_4 (x)(x-6) = 2$$

$$4^2 = x(x-6)$$

$$16 = x^2 - 6x$$

$$0 = x^2 - 6x - 16$$

$$0 = (x-8)(x+2)$$

no log. \rightarrow change to exponential form

Ex. 5 Using log properties expand or condense the following

Expand: $\log_{10} \frac{x^3}{y^6 z}$

$$\log_{10} x^3 - \log_{10} y^6 - \log_{10} z$$

$$3 \log_{10} x - 6 \log_{10} y - \log_{10} z$$

Condense:

$$2 \log_{10} 7 + \log_{10} 3 - 3 \log_{10} 12$$

$$\log_{10} 7^2 + \log_{10} 3 - \log_{10} 12^3$$

$$\log_{10} \frac{7^2 \cdot 3}{12^3} = \log_{10} \frac{147}{1728}$$

3.20 Common Logarithms

Common Logarithms - base 10 logarithms

$\log x = \log_{10} x$ calculator

Change of Base Formula - allows you to write equivalent logarithmic expressions that have different bases.

$$\text{Ex. } \log_5 12 = \log_{10} 12 / \log_{10} 5$$

$$\frac{\log_{10} 12}{\log_{10} 5}$$

$$\begin{array}{l} \text{calc.} \\ \log 12 = \\ \div \log 5 = 1.944 \end{array}$$

$$\log_b a = \frac{\log a}{\log b}$$

Ex. 1 Find Common Logarithms

a. $\log 3 = \log_{10} 3$
 $\approx \boxed{.4771}$

b. $\log .2$
 $\approx -.699$

Ex. 2 Solve Logarithmic Equations Using Exponentiation

If you cannot get the same base, take the log of both sides

$$3^x = 11$$

$$\log 3^x = \log 11$$

$$x \frac{\log 3}{\log 3} = \frac{\log 11}{\log 3}$$

$$x = \frac{\log 11}{\log 3}$$

$$\boxed{x \approx 2.1827}$$

$$4^{x-2} = 11$$

$$\log 4^{x-2} = \log 11$$

$$(x-2) \frac{\log 4}{\log 4} = \frac{\log 11}{\log 4}$$

$$x-2 \approx 1.7297$$

$$+2 \quad +2$$

$$\boxed{x \approx 3.7297}$$

Ex. 3 Solve Exponential Equations Using Logarithms

$$\log_{10} E = 24.55$$

* on calculator $2^{nd} \log \rightarrow 10^{\square}$

$$10^{24.55} = E$$

$$\boxed{E \approx 3.548 \times 10^{24}}$$

Ex. 4 Solve Exponential Inequalities Using Logarithms

$$\log_4 25 = \frac{\log 25}{\log 4} \approx 2.3219$$

Ex. 5 Change of Base Formula

$$\log_2 3 = \frac{\log 3}{\log 2} \approx \boxed{1.5855}$$

3.21 Base e and Natural Logarithms

Natural Base $\boxed{e = 2.7128...}$ (derived from $(1 + 1/n)^n$)

Natural Base Exponential Function - an exponential function with base e

Natural Logarithm - logarithm with base e

Natural Logarithm Function - $y = \ln x$, which is the inverse of $y = e^x$

*** $\ln 1 = 0$ and $\ln e = 1$ *****

$$y = \ln x \text{ and } e^x = y$$

inverses

$$\log_e x = \ln x = \ln x$$

$$y = \log x$$

inverse

$$10^x = y$$

Ex. 1 Evaluate Natural Base Expressions

a.

$$e^2 \approx 7.389$$

b.

$$e^{-18} \approx 1.523 \times 10^{-8}$$

Ex. 2 Evaluate Natural Logarithmic Expressions

a.

$$\ln 4 \approx 1.386$$

b.

$$\ln .05 \approx$$

Ex. 3 Write Equivalent Expressions

a.

$$e^x = 5 \rightarrow \log_e 5 = x \rightarrow \ln 5 = x \quad \left\{ \begin{array}{l} e^{.6931} = x \Rightarrow \log_e x = .6931 \\ \ln x = .6931 \end{array} \right.$$

b.

Ex. 4 Inverse Property of Base e and Natural Logarithms

a.

$$e^{\ln x} = x \quad \ln e^x = x$$

b.

$$e^{\ln 7} = 7$$

$$\ln e^{4x+3} = 4x+3$$

Ex. 5 Solve Base e Equations

$$\begin{array}{r} 5e^{-x} - 7 = 2 \\ +7 \quad +7 \\ \hline 5e^{-x} = 9 \\ \frac{5}{5} \quad \frac{5}{5} \end{array}$$

$$\begin{array}{r} e^{-x} = \frac{9}{5} \\ \ln \frac{9}{5} = -x \\ .588 = -x \\ \underline{-1} \quad \underline{-1} \end{array}$$

$$x = -.588$$

Ex. 6 Solve Base e Inequalities

a.

$$\begin{array}{r} e^x = 30 \\ \ln e^x = \ln 30 \\ x = 3.0 \end{array}$$

b.

$$\begin{array}{r} e^{5x} = 25 \\ \frac{1}{5} \ln 25 = 5x \cdot \frac{1}{5} \\ \frac{1}{5} \ln 25 = x \quad x \approx .644 \end{array}$$

Ex. 7 Solve Natural Log Equations and Inequalities

a.

$$\begin{array}{r} \ln 5x = 4 \\ e^4 = \frac{5x}{5} \\ x \approx 10.92 \end{array}$$

b.

$$\begin{array}{r} \ln (x-1) = -2 \\ e^{-2} = x-1 \\ +1 \quad +1 \\ e^{-2} + 1 = x \quad x \approx 1.135 \end{array}$$

3.22. Exponential Growth and Decay

Rate of decay - when a quantity decreases at a fixed percent/rate each year or other period of time

Rate of growth - when a quantity increases at a fixed percent/rate each year or other period of time

Ex. 1 Exponential Decay of the Form $y = a(1-r)^t$

how long does it take for half material to decay?

$$a = 130$$

$$y = 65$$

$$r = 11\% \text{ decrease/hour}$$

$$t = ?$$

$$\frac{65}{130} = \frac{130}{130} (1 - .11)^t$$

$$.5 = .89^t$$

$$\log .5 = \log .89^t$$

$$\log .5 = t \log .89$$

$$t = \frac{\log .5}{\log .89}$$

a = initial amount
 P = principal / initial amount
 r = rate (in decimal)
 k = constant or rate
 t = time (usually years)
 $y/A/P(t)$ = ending amount

$$t = \frac{\log .5}{\log .89} \quad t \approx 5.95$$

Ex. 2 Exponential Decay of the Form $y = ae^{-kt}$

Half life uranium is 5460 years. Find rate of decay?

$$a = 2g$$

$$y = 1g$$

$$t = 5460$$

$$k = ?$$

$$y = ae^{-kt}$$

$$\frac{1}{2} = \frac{2}{2} e^{-k \cdot 5460}$$

$$.5 = e^{-5460k}$$

$$\ln .5 = \frac{-5460k}{-5460}$$

$$k = \frac{\ln .5}{-5460}$$

$$k \approx 1.27 \times 10^{-4}$$

$$k \approx .000127$$

Ex. 3 Exponential Growth of the Form $y = a(1+r)^t$

$$a = 120,000 \rightarrow 1913$$

$$r = 1.5\% \rightarrow .015$$

find value in 2013

$$y = 120,000(1 + .015)^{100}$$

$$y = 120,000(1.015)^{100}$$

$$y = 531,845.48$$

Ex. 4 Exponential Growth of the Form $y = ae^{kt}$

continuous growth

$$2013 \rightarrow 500$$

$$\text{rate} = k = 2.5\% \rightarrow .025$$

find amount 2025

$$.025(12)$$

$$y = 500e$$

$$y = \$674.93$$