$\frac{0}{\#} = 0 \qquad \frac{\#}{0} = \text{undefined}$

2.7 Solving Rational Equations and Inequalities

Rational Equation- one or more rational expressions with an equal sentence
fraction

Rational Inequalities- one or more rational expressions with an inequality

$\frac{(5)1}{(5)3} + \frac{2(3)}{5(3)} \quad \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$

Ex. 1

a. $x \neq 1$

$\frac{x}{x-1} = \frac{1}{2}$

$2(x) = 1(x-1)$

$2x = x - 1$

$-x \quad -x$

$x = -1$

b. $d \neq -1$, $d \neq 2$

$\frac{2}{d+1} = \frac{1}{d-2}$

$2(d-2) = 1(d+1)$

$2d - 4 = d + 1$

$-d \quad -d$

$1d - 4 = 1$

$+4 \quad +4$

$d = 5$

$\frac{2}{6} = \frac{1}{3}$

Ex. 2

a. $x \neq 0$, $x \neq -2$

$\frac{15}{x} + \frac{9x-7}{(x+2)} = \frac{9}{1}$

$\frac{x(x+2)}{1}\frac{15}{\cancel{x}} + \frac{9x-7}{\cancel{(x+2)}}\frac{(x)(x+2)}{1} = 9\frac{(x)(x+2)}{1}$

$15(x+2) + (9x-7)(x) = 9x(x+2)$

$15x + 30 + 9x^2 - 7x = 9x^2 + 18x$

$8x + 30 + 9x^2 = 9x^2 + 18x$

$-9x^2 \quad -9x^2$

$8x + 30 = 18x$

$-8x \quad -8x$

$\frac{30}{10} = \frac{10x}{10}$

$3 = x$

b. $w \neq -2$, $w \neq 2$

$\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$

$\frac{(w+2)(w-2)}{1}\frac{1}{w+2} + \frac{(w+2)(w-2)}{1}\frac{1}{w-2} = \frac{4}{w^2-4}\,(w+2)(w-2)$

$w - 2 + w + 2 = 4$

$\frac{2w}{2} = \frac{4}{2}$

$w = 2$

no solution $\varnothing$

cw
p516
33,34,35
p510
11,13,19,
23

Ex. 3 ~~Word~~ Problems with Rational Equations

$t \neq 3$

$\frac{9}{t-3} = \frac{t-4}{t-3} + \frac{1}{4}$

$4(\cancel{t-3})\frac{9}{\cancel{t-3}} = 4(\cancel{t-3})\frac{(t-4)}{\cancel{t-3}} + \frac{1}{\cancel{4}}(4)(t-3)$

$4(9) = 4(t-4) + t - 3$

$36 = 4t - 16 + t - 3$

$36 = 5t - 19$

$+19 \quad +19$

$\frac{55}{5} = \frac{5t}{5}$

$11 = t$

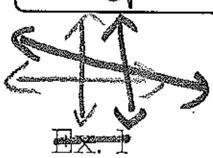
2.8 Solving Systems of Equations Algebraically

$y = mx + b$
↓ slope
↓ y intercept $(0, b)$

<u>System of Equations</u>- two or more equations with the same variables

Systems of Linear Equations

- **Consistent**
  - **Independent** CI

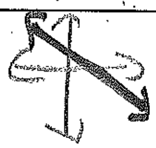
    

    intersecting lines
    solution (x, y)

    ~~Ex. 1~~

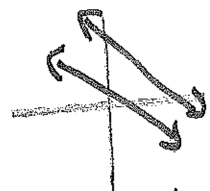
    $y = 2x + 3$
    $y = 3x - 1$

  - **Dependent** CD

    coincidal
    same line
    ∞ solutions
    same equations
    $y = 2x + 6$
    $-3y = -6x - 18$
    solve: get something that is <u>ALWAYS</u> true

- **Inconsistent** I

  parallel lines
  do <u>not</u> intersect
  No solution
  (no points in common

  → have same slope
  different y intercepts
  $y = \boxed{2}x + \underline{6}$
  $y = \boxed{2}x - \underline{1}$

---

ex1 solving using elimination

~~Ex. 2~~

$2x + y = 5$
$x - y = 1$
$\frac{3x}{3} = \frac{6}{3}$
$\boxed{x = 2}$
$2(2) + y = 5$
$4 + y = 5$
$-4 \quad -4$
$\boxed{y = 1}$

~~Ex. 3~~

$(2, 1)$

ex2. solve using Substitution

$x + 2y = 8 \rightarrow x = 8 - 2y$
$\frac{1}{2}x - y = 18$
$\frac{1}{2}(8 - 2y) - y = 18$
$4 - 1y - y = 18$
$4 - 2y = 18$
$-4 \quad -4$
$\frac{-2y}{-2} = \frac{14}{-2}$
$\boxed{y = -7}$
$x = 8 - 2(-7)$
$x = 8 + 14$
$\boxed{x = 22}$
$(22, -7)$

ex3

$4a + 2b = 15$
$m(-1)\ 2a + 2b = 7$

$4a + 2b = 15$
$-2a - 2b = -7$
$\frac{2a}{2} = \frac{8}{2}$
$\boxed{a = 4}$
$2(4) + 2b = 7$
$8 + 2b = 7$
$-8 \quad -8$
$\frac{2b}{2} = \frac{-1}{2}$
$\boxed{b = -\frac{1}{2}}$
$(4, -\frac{1}{2})$

Ex.4 Compare the quantity in Column A and the quantity in Column B. Then determine whether:

A. The quantity in Column A is greater
B. The quantity in Column B is greater
C. The two quantities are equal
D. The relationship cannot be determined from the information given

i. $2x + y = 11$ → $2x + y = 11$
ii. $x + 3y = 13$ m(-2) → $-2x - 6y = -26$

$-5y = -15$

$\frac{-5y}{-5} = \frac{-15}{-5}$

$\boxed{y = 3}$

$x + 3(3) = 13$

$x + 9 = 13$

$-9 \quad -9$

$x = 4$

Column A	Column B
$X = 4$	$Y = 3$

A

Ex. 5

$3x - 7y = -14$ m(2) → $6x - 14y = -28$

$5x + 2y = 45$ m(7) → $35x + 14y = 315$

$41x = 287$

$\frac{41x}{41} = \frac{287}{41}$

$\boxed{x = 7}$

$5(7) + 2y = 45$

$35 + 2y = 45$

$-35 \quad -35$

$2y = 10$

$\frac{2y}{2} = \frac{10}{2}$

$\boxed{y = 5}$

(7, 5)

Ex. 6

$8x + 2y = 17$ → $8x + 2y = 17$

$-4x - y = 9$ m(2) $-8x - 2y = 18$

$0 \neq 35$

no

no solution

parallel lines

Ex.7

$9x - 6y = 24 \quad m(2) \rightarrow \quad 18x - 12y = 48$

$6x - 4y = 16 \quad m(-3) \rightarrow -18x + 12y = -48$

$0 = 0$

always true

∞ solutions

CW. p146 # 15, 19

p120 # 13, 17, 19, 32

Ex. 8 Variety of Systems

$y = x^2 + 4x - 2$

$y = \boxed{2x+1}$

$2x + 1 = x^2 + 4x - 2$

$-2x \quad\quad -2x$

$1 = x^2 + 2x - 2$

$-1 \quad\quad\quad -1$

$0 = x^2 + 2x - 3$

$0 = (x - 1)(x + 3)$

$-1x$

$+3x$

$x - 1 = 0 \qquad x + 3 = 0$

$+1 \ \ +1 \qquad -3 \ \ -3$

$\boxed{x = 1} \qquad \boxed{x = -3}$

$x = 1$

$y = 2(1) + 1$

$y = 2 + 1$

$y = 3$

$x = -3$

$y = 2(-3) + 1$

$y = -6 + 1$

$y = -5$

$(1, 3) \quad (-3, -5)$

Ex. 9 Variety of Systems

$m(-2) \quad x^2 + y^2 = 25 \rightarrow -2x^2 - 2y^2 = -50$

$3x^2 + 2y^2 = 59 \qquad 3x^2 + 2y^2 = 59$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

$x = 3 \quad x = -3$

$x = 3$

$(3)^2 + y^2 = 25$

$9 + y^2 = 25$

$-9 \quad\quad -9$

$\sqrt{y^2} = \sqrt{16}$

$y = 4 \quad y = -4$

$x = -3$

$(-3)^2 + y^2 = 25$

$9 + y^2 = 25$

$-9 \quad\quad -9$

$\sqrt{y^2} = \sqrt{16}$

$y = 4 \quad y = -4$

$(3, 4) \ (3, -4) \ (-3, 4) \ (-3, -4)$

2.9 Solving Systems of Equations in Three Variables

Ordered Triple- The solution of a system of equations with three variables (x, y, z)

Systems of Equations in Three Variables

One Solution | Infinite Solutions | No Solution

Ex. 1 One Solution

$$\begin{cases} x+2y+z=10 \\ 2x-y+3z=-5 \\ 2x-3y-5z=27 \end{cases}$$

m(-3) →

$-3x-6y-3z=-30$
$2x-y+3z=-5$
$\boxed{-1x-7y=-35}$

m5

$5x+10y+5z=50$
$2x-3y-5z=27$
$\boxed{7x+7y=77}$

$-x-7y=-35$
$7x+7y=77$
$\frac{6x}{6}=\frac{42}{6}$
$\boxed{x=7}$

$(7,4,-5)$

$7(7)+7y=77$
$49+7y=77$
$-49 \quad -49$
$\frac{7y}{7}=\frac{28}{7}$
$\boxed{y=4}$

$7+2(4)+z=10$
$7+8+z=10$
$15+z=10$
$-15 \quad -15$
$\boxed{z=-5}$

Ex. 2 Infinite Solutions

$4x-6y+4z=12$ m(3)
$6x-9y+6z=18$ m(-2)
$5x-8y+10z=20$

→ $12x-18y+12z=36$
$-12x+18y-12z=-36$
$0=0$
always true

∞ solutions

Ex. 3 No Solution

$$
\begin{aligned}
6a + 12b - 8c &= 24 \\
9a + 18b - 12c &= 30 \\
4a + 8b - 7c &= 26
\end{aligned}
$$

$m(3) \rightarrow 18a + 36b - 24c = 72$

$m(-2)$ $\quad -18a - 36b + 24c = -60$

$$0 \neq 12$$

no solution

Classwork

p142 6-8