2.4 Dividing Polynomials

Ex. 1 Divide a Polynomial by a Monomial

$$\frac{4x^3y^2 + 8xy^2 - 2x^2y^3}{4xy}$$

$$\frac{4x^3y^2}{4xy} + \frac{8xy^2}{4xy} - \frac{2x^2y^3}{4xy}$$

$$x^2y + 2y - \frac{xy^2}{2}$$

long division from 4th grade

$1255 \div 5$

$$\begin{array}{r} 251 \\ 5\overline{)1255} \\ -10 \\ \hline 25 \\ -25 \\ \hline 05 \\ -5 \\ \hline 0 \end{array}$$

factor

0 → remainder

Ex. 2 Division Algorithm (using long division)

a) $(x^2+7x+12) \div (x+3)$

$$\begin{array}{r} x+4 \\ x+3\overline{)x^2+7x+12} \\ -x^2-3x \\ \hline 4x+12 \\ -4x-12 \\ \hline 0 \end{array}$$

b) $(x^2-8x-48) \div (x-12)$

$$\begin{array}{r} x+4 \\ x-12\overline{)x^2-8x-48} \\ -x^2+12x \\ \hline 4x-48 \\ -4x+48 \\ \hline 0 \end{array}$$

c) $\dfrac{(4x^4-x^3-19x^2+11x-2)}{x-2}$

$$\begin{array}{r} 4x^3+7x^2-5x+1 \\ x-2\overline{)4x^4-x^3-19x^2+11x-2} \\ -4x^4+8x^3 \\ \hline 7x^3-19x^2 \\ -7x^3+14x^2 \\ \hline -5x^2+11x \\ +5x^2-10x \\ \hline x-2 \\ -x+2 \\ \hline 0 \end{array}$$

Ex. 3 Quotient with a Remainder (using long division)

$(t^2+3t-9) \div (5-t) \rightarrow (-t+5)$

$$\begin{array}{r} -t-8 + \dfrac{31}{-t+5} \\ -t+5\overline{)t^2+3t-9} \\ -t^2+5t \\ \hline 8t-9 \\ -8t+40 \\ \hline 31 \end{array}$$

31 remainder

you must divide by a binomial where the coefficient of the variable is 1. $\boxed{x-r} \rightarrow$ r goes on the shelf

Ex. 4 Synthetic Division

a. $(5x^3-13x^2+10x-8)\div(x-2)$

$$\begin{array}{r|rrrr} 2 & 5 & -13 & 10 & -8 \\ & & 10 & -6 & 8 \\ \hline & 5 & -3 & 4 & 0 \end{array}$$

0 remainder

$$\boxed{5x^2-3x+4}$$

b. $(3x^4+6x^3-2x+4)\div(x+2)$

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -2 & 4 \\ & & -6 & 0 & 0 & 4 \\ \hline & 3 & 0 & 0 & -2 & 8 \end{array}$$

8 ← remainder

$3x^3+0x^2+0x-2+\frac{8}{x+2}$

$$\boxed{3x^3-2+\frac{8}{x+2}}$$

Ex. 5 Divisor with Coefficient other than 1

$$\left(\frac{8x^4}{2}-\frac{4x^2}{2}+\frac{x}{2}+\frac{4}{2}\right)\div\left(\frac{2x}{2}+\frac{1}{2}\right)$$

$$\left(4x^4-2x^2+\tfrac{1}{2}x+2\right)\div\left(x+\tfrac{1}{2}\right)$$

$$\frac{2}{1}\frac{(2)}{(2)}-\frac{1}{2}=\frac{4}{2}-\frac{1}{2}=\frac{3}{2}$$

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 4 & 0 & -2 & \frac{1}{2} & 2 \\ & \downarrow & -2 & +1 & +\frac{1}{2} & -\frac{1}{2} \\ \hline & 4 & -2 & -1 & 1 & \frac{3}{2} \end{array}$$

→ remainder

$$4x^3-2x^2-x+1+\frac{\frac{3}{2}}{x+\frac{1}{2}} \rightarrow \frac{\frac{3}{2}\left(\frac{2}{1}\right)}{\left(\frac{2}{1}\right)x+\frac{1}{2}\left(\frac{2}{1}\right)} = \frac{3}{2x+1}$$

$$4x^3-2x^2-x+1+\frac{3}{2x+1}$$

hw p236
#22-36 even
synthetic ÷

$ax^2+bx+c=0$ quad. formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

2.5 The Remainder and Factor Theorems

<u>The Remainder Theorem</u>- the value of f(x) is the same as the remainder when divided by x

<u>Synthetic Substitution-</u> when synthetic division is used to evaluate a function

<u>Depressed Polynomial</u>- the quotient of a polynomial and a binomial factor

<u>Factor Theorem</u>- when you are using synthetic division and get a remainder of zero that binomial is a factor of the polynomial

56 → factors 8, 7 ex.
$x^2+7x+12$ → factors $(x+3)(x+4)$

Ex. 1 Synthetic Substitution

$f(x)=2x^4-5x^2+8x-7$ find $f(6)$

$(x-6)$

traditional substitution

$f(6)=2(6^4)-5(6)^2+8(6)-7$
$=2453$

synthetic ÷

6	2	0	-5	8	-7
	↓	12	72	402	2460
	2	12	67	410	2453

Ex. 2 Use the Factor Theorem.

Show $(x+3)$ is a factor of x^3+6x^2-x-30

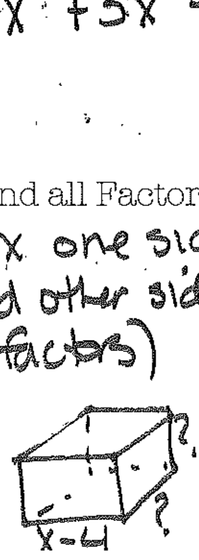
-3	1	6	-1	-30
	↓	-3	-9	30
	1	+3	-10	0 ← factor

$x^2+3x-10$ → depressed polynomial

hw..
p368 #21, 23, 25
p376
25 (-2)
26 (4)
28 (7)
29 ($-\frac{3}{2}$)

Ex. 3 Find all Factors of a Polynomial

Box one side $(x-4)$ volume $v(x)=x^3+3x^2-36x+32$
find other sides (factors)



4	1	3	-36	32
	↓	4	28	32
	1	7	-8	0

x^2+7x-8 → depressed eq.

$\boxed{(x-1)(x+8)(x-4)}$

2.6 Roots and Zeros = answer = x intercept

Fundamental Theorem of Algebra – every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.

Ex. 1 Determine Number and Type of Roots

Solve each equation. State the number and type of roots.

a. $x+3=0$

$-3 \quad -3$

$x=-3$

1 root/zero/solution

1 Real root

b. $x^2-8x+16=0$

ac ±
16 −8
1, 16
2, 8
4, 4

$(x-4)(x-4)=0$

$-4x \quad -4x$

$x-4=0$ $\quad$ $x-4=0$

$+4 \; +4$ $\quad$ $+4 \; +4$

$x=4$ $\quad$ $x=4$

double real root

c. $x^3+2x=0$

$x(x^2+2)=0$

$x=0$ $\quad$ $x^2+2=0$

$-2 \; -2$

$\sqrt{x^2}=\sqrt{-2}=\sqrt{-1\cdot 2}$

$x=\pm i\sqrt{2}$

$\boxed{x=0 \quad x=i\sqrt{2} \quad x=-i\sqrt{2}}$

3 roots

1 real & 2 imaginary

d. $x^4-1=0$

$(x^2)^2-(1)^2=0$

$(x^2+1)(x^2-1)=0$

$x^2+1=0$ $\quad$ $x^2-1=0$

$-1 \; -1$ $\quad$ $+1 \; +1$

$\sqrt{x^2}=\sqrt{-1}$ $\quad$ $\sqrt{x^2}=\sqrt{1}$

$x=\pm i$ $\quad$ $x=\pm 1$

$\boxed{x=i \quad x=-i \quad x=1 \quad x=-1}$

4 roots

2 imaginary & 2 real

Corollary to Fundamental Theorem- A polynomial equation $P(x) = 0$ of degree n with complex coefficients has exactly n roots in the set of complex numbers.

the degree of equation = number of roots

$x-a \rightarrow$ opposite of a $\quad x=b \rightarrow$ use b

Ex. 2 Use Synthetic Division to Find Zeros

$f(x)=x^3-7x^2+2x+40$; $(x-5)$ $\quad x=5$

5	1	-7	2	40
	↓	5	-10	-40
	1	-2	-8	0

$x^2-2x-8=0$ $\quad$ ax: -8; +: -2; $1,8$; $4,2$

$(x-4)(x+2)=0$ $\quad -4x$, $+2x$

$x-4=0$, $+4\ +4$, $x=4$ $\quad x+2=0$, $-2\ -2$, $x=-2$

$x=-2$
$x=4$
$x=5$

$f(x)=x^3-4x^2+6x-4$

one of zeros is 2 $\quad x=2$

2	1	-4	6	-4
	↓	2	-4	4
	1	-2	2	0

$x^2-2x+2=0$ $\quad$ ax: 2; +: -2; $1,2$

$a=1$ $b=-2$ $c=2$

$$x=\frac{--2\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{4-8}}{2}=\frac{2\pm\sqrt{-4}}{2} \rightarrow 2i$$

$$x=\frac{2}{2}\pm\frac{2}{2}i \quad x=1\pm i$$

$x=2$
$x=1+i$
$x=1-i$

Ex. 4 Use Zeros to Write a Polynomial Function

to find possible zeros:

Rule: $\dfrac{\pm\text{ last term factors}}{\text{first term factors}}$

$f(x)=3x^2+2x-1$

last term $\rightarrow 1 \rightarrow 1$

first term $\rightarrow 3 \rightarrow 1,3$

possibilities $\dfrac{\pm 1}{1,3}=\pm\dfrac{1}{1},\pm\dfrac{1}{3}=+1,-1,+\dfrac{1}{3},-\dfrac{1}{3}$

★ imaginary roots come in conjugate pairs

ex. given zeros: 2, 3i find polynomial

$x=2$, $-2\ -2$, $(x-2)=0$ $\quad x=3i$, $-3i\ -3i$, $(x-3i)=0$ $\quad$ ★ $x=-3i$, $+3i\ +3i$, $(x+3i)=0$

$(x-2)(x-3i)(x+3i)=0$

$(x-2)(x^2+3xi-3xi-9i^2)=0$ $\quad -9\cdot -1$

$(x-2)(x^2+9)=0$

$x^3+9x-2x^2-18=0$

$\boxed{x^3-2x^2+9x-18=0}$

cw. 2.6
p402 #30-35
state # zeros.
p403 #36(4)
#41 $(2,\frac{1}{2})$