BL3

Unit 2 Notes

1.1 Solving Basic Equations and Inequalities

Properties of Equality

PROPERTY	SYMBOLS	EXAMPLE
REFLEXIVE	A = A	-7 + N = -7 + N
SYMMETRIC	A = B then B = A	5 + 4X = 17, 17 = 5 + 4X
TRANSITIVE	IF A = C & B = C, THEN A = B	2X +1= 7 & 7 = 5X -8, THEN 2X+1 = 5X -8
SUBSTITUTION	IF A = B, then A may be replaced by B, and B may be replaced by A	M + N If M= 3, and N = 4 Then 3 + 4 = 7
ADDITION/SUBTRACTION		
MULTIPLICATION/DIVISION		

Ex. 1 Solve One-Step Equations

a. $\frac{5}{4}\cdot\frac{4}{5}x = 20\cdot\frac{5}{4}$

$x = 25$

c. $(\sqrt{x})^2 = (5)^2$

$x = 25$

b. $\sqrt{x^2} = \sqrt{9}$

$x = 3 \quad x = -3$

$x = \pm 3$

d. $\sqrt{x^2} = \sqrt{\frac{-25}{16}}$

$x = \pm\sqrt{\frac{-1(55)}{4\cdot 4}}$

$x = \pm\frac{5i}{4}$

CLT = combine like terms

Ex. 2 Solve a Multi-Step Equation

a. $2(2x+3) - 3(4x-5) = 22$

distributive

$4x + 6 - 12x + 15 = 22$

CLT

$-8x + 21 = 22$

$-21 \quad -21$

$\frac{-8x}{-8} = \frac{1}{-8}$

$x = -\frac{1}{8}$

c. $\frac{3x^2}{3} = \frac{81}{3}$

$\sqrt{x^2} = \sqrt{27}$

$x = \pm\sqrt{3 \cdot 3 \cdot 3}$

$x = \pm 3\sqrt{3}$

b. $(\sqrt{2x})^2 = (4)^2$ square

$\frac{2x}{2} = \frac{16}{2}$ divide

$x = 8$

d. $\sqrt{(x+4)^2} = \sqrt{-25}$

$(x+4) = \pm\sqrt{-1 \cdot 5 \cdot 5}$

$x + 4 = \pm 5i$

$-4 \quad -4$

$x = -4 \pm 5i$

Ex. 3 Solve for a Variable

a. $\frac{2}{1}A = \frac{(b_1 + b_2)}{2}h$ for b_1

$\frac{2A}{h} = \frac{(b_1 + b_2)h}{h}$

$\frac{2A}{h} = b_1 + b_2$

$-b_2 \quad -b_2$

$\frac{2A}{h} - b_2 = b_1$

b. $\frac{3}{4}V = \frac{4}{3}\pi r^3 \cdot \frac{3}{4}$ for r

$\frac{1}{\pi}\frac{3V}{4} = \pi r^3 \cdot \frac{1}{\pi}$

$\sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{r^3}$

$\sqrt[3]{\frac{3V}{4\pi}} = r$

Ex. 4 Write an Equation

total budget \$1685

small improvements \$425

if we want to replace 6 doors, how much for each door?

Let x = cost of a door

$1685 = 425 + 6x$

$-425 \quad -425$

$\frac{1260}{6} = \frac{6x}{6}$

\$210 = x

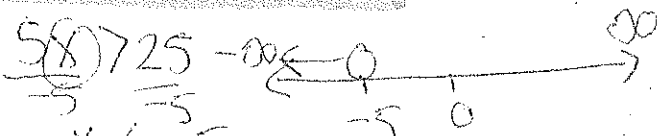
Solving Inequalities

Same Equality rules apply to solving for inequalities as solving equations.

When multiplying or dividing by a negative number, the direction of the inequality changes

$\frac{-5x}{-5} > \frac{25}{-5}$

$x < -5$



$\{x \mid x < -5\}$

Set Builder Notation: {x|x...}

Interval Notation: The endpoints of the solution to an inequality written in [ ]

answer is for sure, =

( ) answer is not for sure, ≠

$(-\infty, -5)$

Ex. 5 Solving Using +/-

$x + 4 < -12$

$-4 \quad -4$

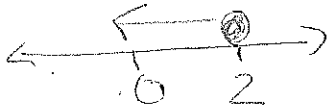
$x < -16$

$(-\infty, -16)$ $\{x \mid x < -16\}$

Ex. 6 Solving using x/÷

$\frac{-3x}{-3} \geq \frac{-6}{-3}$

$x \leq 2$



$(-\infty, 2]$ $\{x \mid x \leq 2\}$

Ex. 7 Solve a Multi-Step Inequality

$(\sqrt{2x+5})^2 < (2)^2$

$2x + 5 < 4$

$-5 \quad -5$

$\frac{2x}{2} < \frac{-1}{2}$

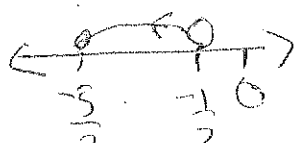
$x < -\frac{1}{2}$

$* \quad 2x + 5 \geq 0$

$-5 \quad -5$

$\frac{2x}{2} \geq \frac{-5}{2}$

$x \geq -\frac{5}{2}$



$\left[-\frac{5}{2}, -\frac{1}{2}\right]$

Ex. 8 Write an Inequality

2000 lbs max elevator capacity

Craig weighs 160 lbs

each box weighs 64 lbs

how many boxes can Craig safely carry

Let x = # boxes

$160 + 64x \leq 2000$

$-160 \quad -160$

$\frac{64x}{64} \leq \frac{1840}{64}$

$x \leq 28.75$

28 boxes

(-1, 2] open closed

9/9 BL2

2.2 Solving Absolute Value Equations & Inequalities

Absolute Value: the distance of that number to zero on the number line. distance is always positive | |

Empty Set: No solution or { }, ∅ { }

Ex. 1 Evaluate an Expression w/ Abs. Value

$1.4 + |5y - 7|$ if $y = -3$

$1.4 + |5(-3) - 7|$

$1.4 + |-15 - 7|$

$1.4 + |-22|$

$1.4 + 22$

$= 23.4$

$22 + 1.4 = 23.4$

Ex. 2 Solve an Abs. Value Equation.

a. $|x| = 3$

$x = 3 \quad x = -3$

$x = \pm 3$

b. $|x - 18| = 5$

$x - 18 = 5$ $\quad x - 18 = -5$

$+18 \quad +18 \quad +18 \quad +18$

$x = 23 \quad x = 13$

Ex. 3 No Solution

$|5x - 6| + 9 = 0$

$-9 \quad -9$

$|5x - 6| = -9$

$\emptyset$

Ex. 4 One Solution

$|x + 6| = x - 7$

$x + 6 = x - 7$ $\quad x + 6 = -(x - 7)$

$-6 \quad -6 \quad x + 6 = -x + 7$

$x = x - 13 \quad -6 \quad -6$

$-x \quad -x \quad x = -x + 1$

$0 \neq -13 \quad +x \quad +x$

$2x = 1$

$x = \frac{1}{2}$

Compound Inequality: Two inequalities joined by the word *and* or the word *or*. Each inequality is solved (two solutions)

And Inequalities: Is true if and only if both of the individual inequalities are true (graph will meet/intersect) $<$ $\leq$ usually in the middle

Or Inequalities: is true if one or more of the inequalities is true $>$ $\geq$ usually at the ends

Ex. 5 "and" Compound Inequality

$13 < 2x+7 \leq 17$

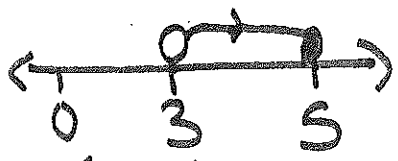
$13 < 2x+7$ and $2x+7 \leq 17$

$-7 \quad -7 \qquad\qquad -7 \quad -7$

$6 < 2x \qquad\qquad 2x \leq 10$

$\frac{6}{2} < \frac{2x}{2} \qquad\qquad \frac{2x}{2} \leq \frac{10}{2}$

$3 < x \qquad\qquad x \leq 5$



$(3, 5]$

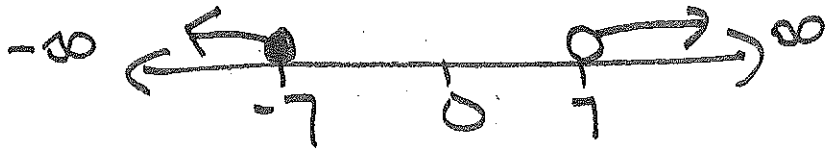
Ex. 6 "or" compound Inequality

$y-4 > 3$ or $y+4 \leq -3$

$+4 \quad +4 \qquad\qquad -4 \quad -4$

$y > 7 \qquad\qquad y \leq -7$

$(-\infty, -7]$ or $(7, \infty)$



Absolute Value Inequalities

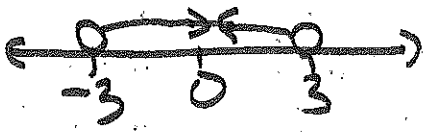
** If $|x| < b$, then your answer is similar to an *and* inequality, where $-b < a < b$

**If $|x| > b$, then your answer is similar to an *or* inequality, where $a > b$ or $a < -b$

Ex. 7 Abs. Value Inequality (<) Less Than(d)

$|x| < 3$

$x < 3$ and $x > -3$ $\qquad (-3, 3)$



Ex. 8 Abs. Value Inequality (>) Great(or)

$|x+5| > 6$

$x+5 > 6$ or $x+5 < -6$

$-5 \quad -5 \qquad\qquad -5 \quad -5$

$x > 1 \qquad\qquad x < -11$

$(-\infty, -11)$ or $(1, \infty)$

Ex. 9 Solve a Multi-Step Absolute Value Inequality

$|3x-12| \geq 6$

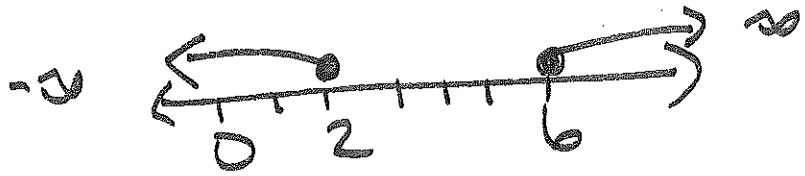
$3x-12 \geq 6$ or $3x-12 \leq -6$

$+12 \quad +12 \qquad +12 \quad +12$

$\frac{3x}{3} \geq \frac{18}{3} \qquad \frac{3x}{3} \leq \frac{6}{3}$

Ex. 10 Write an Absolute Value Inequality

$x \geq 6$ or $x \leq 2$



$(-\infty, 2]$ or $[6, \infty)$

B L3

2.3 Factoring Polynomials

Factoring Techniques

m ? of 'erms	Factoring Technique	General Case
number	Greatest common factor GCF	$a^3b^3 + 2a^2b - 4ab^2 = ab(a^2b^2+2a-4b)$
Two	Difference of 2 Squares	$(a)^2-(b)^2 = (a+b)(a-b)$
	Sum of 2 Cubes	$(a)^3+(b)^3 = (a+b)(a^2-ab+b^2)$
	Difference of 2 Cubes SOAP	$(a)^3-(b)^3 = (a-b)(a^2+ab+b^2)$
Three	Perfect Square Trinomials	$a^2+2ab+b^2 = (a+b)^2$
		$a^2-2ab+b^2 = (a-b)^2$
	General Trinomials	$acx^2+(ad+bc)x+bd=(ax+b)(cx+d)$
4 or more	Grouping	$ax+bx+ay+by=x(a+b)+y(a+b)=(a+b)(x+y)$

Ex. 1 GCF $6x^2y^2 - 2xy^2 + 6x^3y = 2xy(3xy - 1y + 3x^2)$

Ex. 2 Grouping $a^3 - 4a^2 + 3a - 12$

$a^2(a-4) + 3(a-4)$

$(a^2+3)(a-4)$

ax^2+bx+c $\quad a\times c \quad \pm b$

Ex. 3 Two or Three Terms

a. $x^2+7x+12$

$(x+3)(x+4)$

$+3x$, $+4x$

$x^2+7x+12$ $\quad a\times c: 12 \quad \pm: +7$

$3,4$

$x^2+3x+4x+12$

$x(x+3)+4(x+3)$

$(x+4)(x+3)$

b. $5x^2-13x+6$ $\quad a\times c: 30 \quad \pm: -13$

$5,6$; $1,30$; $\cancel{2,15}$; $\cancel{3,10}$

$5x^2-3x-10x+6$

$x(5x-3)-2(5x-3)$

$(x-2)(5x-3)$

$5x^2-13x+6$ $\quad 1,6$; $2,3$

$(5x-3)(x-2)$

$-3x$, $-10x$

c. $25x^2-49y^2$ $\quad a^2-b^2=(a+b)(a-b)$

$(5x)^2-(7y)^2$

$a \quad b$

$(5x+7y)(5x-7y)$

d. a^3+8 $\quad a^3+b^3=(a+b)(a^2-ab+b^2)$

$(a)^3+(2)^3$

$a \quad b$

$(a+2)(a^2-2a+4)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

e. c^3d^3-27

$(cd)^3-(3)^3$

$a \quad b$

$(cd-3)(c^2d^2+3cd+9)$

f. y^4-16 $\quad a^2-b^2=(a+b)(a-b)$

$(y^2)^2-(4)^2$

$a \quad b$

$(y^2+4)(y^2-4)$

$(y^2+4)((y)^2-(2)^2)$

$a \quad b$

$(y^2+4)(y+2)(y-2)$